Reflections on the Gaussian Broadcast Channel: Progress and Challenges

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OUTLINE

Broadcast Channels: Introduction.

The Gaussian scalar broadcast channel.

- ∗ converse via I-MMSE & challenges.

Vector (MIMO) Gaussian broadcast channels.

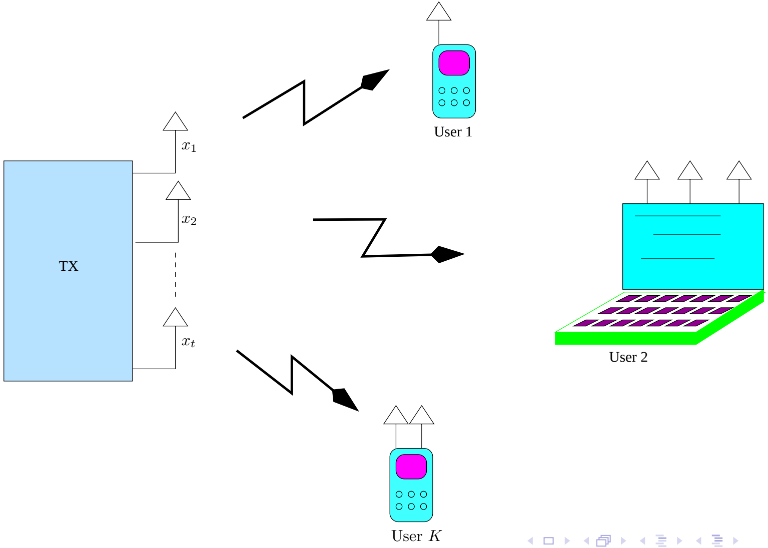
- ∗ historical perspective, applications & challenges.

Broadcast channels – a network motivated outlook.

Concluding remarks.

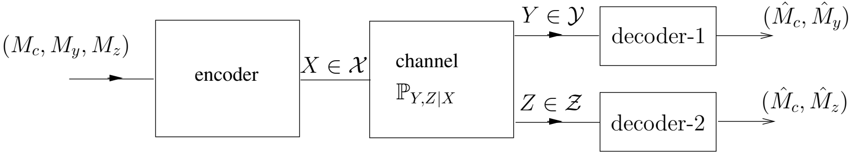
References.

A Broadcast Channel



- Historical Perspective:

  T. M. Cover, “Broadcast Channels,” IEEE Trans. Inform. Theory, vol. IT–18, no. 1, pp. 2–14, January 1972.



(M_c, M_y, M_z) common/private messages.

$X \in \mathcal{X}$ channel input: subjected to input constraints, e.g. $E(X^2) \leq P$.

$Y \in \mathcal{Y}$, $Z \in \mathcal{Z}$ – channel outputs.

Classical Results Review

- (R_c, R_y, R_z) – Information rate triplet.
- Capacity Region in general ???
  - depends on marginals $\mathbb{P}_{Y|X}, \mathbb{P}_{Z|X}$.
- Some solved cases
  - degraded channels [Bergmans, IT'73], [Gallager, PPI'74],
  - less noisy [Körner-Marton, Coll-IT'75],
  - more-capable [El Gamal, IT'79],
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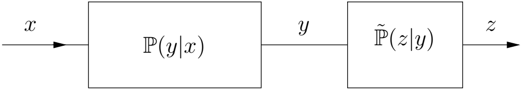


$$\mathbb{P}(z|x) = \int dy\, \mathbb{P}(y|x)\tilde{\mathbb{P}}(z|y)$$

$\tilde{\mathbb{P}}(z|y) = \mathbb{P}(z|y) \Longrightarrow$ physically degraded

$$\mathbb{P}(y,z|x) = \mathbb{P}(y|x)\mathbb{P}(z|y)$$

Degraded Broadcast Channels

- Capacity Region:

  [Bergmans IT'73], [Gallager, PPI'74], [Ahlswede-Körner, IT'75]

  $\Longrightarrow$ Optimize Marton with (Marton's notations): $W, V = \phi,\ U = X$.

  (R_c, R_y, R_z) – satisfying (U – is kept for tradition):

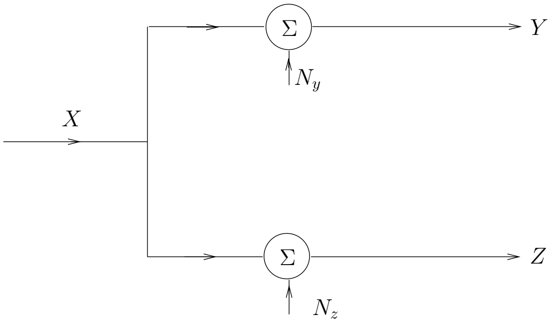
$$R_c + R_z \leq I(U;Z)$$
$$R_y \leq I(X;Y|U)$$

  for some:

$$\mathbb{P}_{U,X,Y,Z} = \mathbb{P}_U \mathbb{P}_{X|U} \mathbb{P}_{Y,Z|X}\,.$$

- set convex, and cardinality constraints $|\mathcal{U}| \leq \min\{|\mathcal{X}|, |\mathcal{Y}_1|, |\mathcal{Y}_2|\}$ in finite alphabets.

GAUSSIAN SCALAR BROADCAST CHANNEL



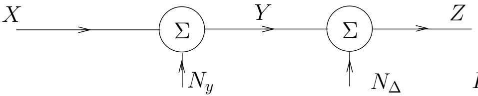
$$Y = X + N_y\,, \quad Z = X + N_z$$

$$E(X^2) \leq P\,, \quad E(N_y^2) = \sigma_y^2,\, E(N_z^2) = \sigma_z^2 \geq \sigma_y^2\,.$$

Gaussian Scalar Broadcast Channel

$\Downarrow$

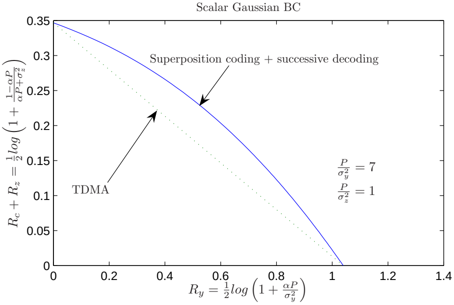
degraded



$$E(N_\Delta^2) = \sigma_z^2 - \sigma_y^2.$$

Capacity Region

$$R_y \leq \frac{1}{2} \log\left(1 + \frac{\alpha P}{\sigma_y^2}\right)$$
$$\bar{R}_z \triangleq R_c + R_z \leq \frac{1}{2} \log\left(1 + \frac{(1-\alpha)P}{\alpha P + \sigma_z^2}\right),$$
$$0 \leq \alpha \leq 1\,.$$



- Achievability by superposition coding [Cover '72].

  $X = X_z + X_y$ superposition coding, $E(X_z^2) = (1-\alpha)P\,,\ E(X_y^2) = \alpha P$.

  $X_z = X_{zc} + X_{zz}$ – carries the messages (M_c, M_z), X_y – carries the message (M_y).

  @ receiver $z \Longrightarrow$ noise level: $\alpha P + \sigma_z^2 \Longrightarrow$ decodes (M_c, M_z).

  @ receiver $y \Longrightarrow$ decodes (M_c, M_z) and strips out $X_z \Longrightarrow$ noise level: $\sigma_y^2 \Longrightarrow$ decodes (M_y).

- superposition: interference removed @ receiver y.

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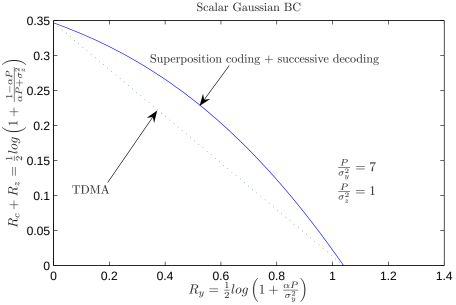

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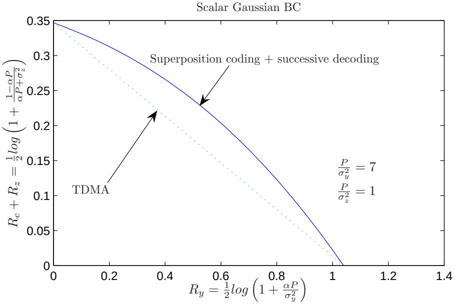

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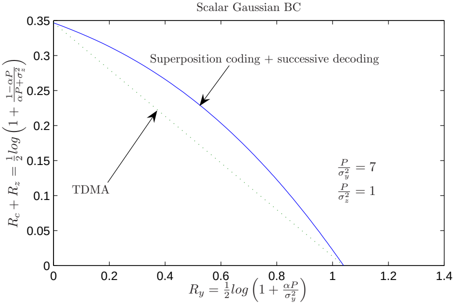

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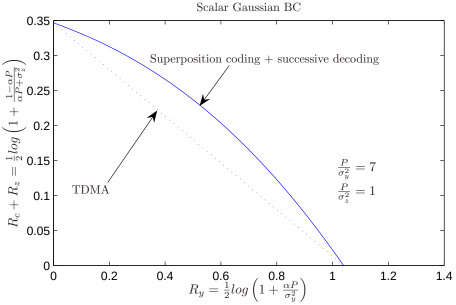

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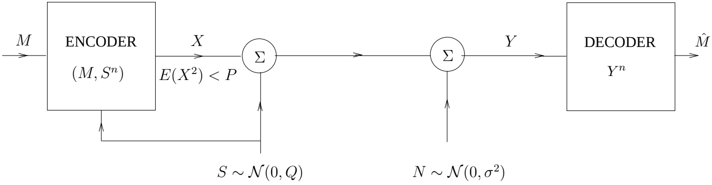
Dirty Paper Coding (DPC)



- state $\{S^n\}$ available un-causally @ transmitter.

  [Gelfand-Pinsker, PCIT'80] – coding idea: binning.

- $C = I(U : Y) - I(U : S)\,, \qquad P_{U,X,S,Y}; U - (X, S) - Y$

Dirty Paper Coding (DPC)

- Dirty Paper: [Costa, IT'83]:

$$U = X + \alpha S\,, \qquad X \perp\!\!\!\perp S\,, \qquad \alpha = \frac{P}{P+N}$$
$$\Longrightarrow C = \frac{1}{2}\log\left(1 + \frac{P}{\sigma^2}\right)$$

- Extended to vectors $(\mathbf{X}, \mathbf{S}, \mathbf{N}, \mathbf{Y})$

  [Yu-Sutivong-Julian-Cover-Chiang, ISIT'01].

- Practical aspects of DP coding [Erez-Shamai-Zamir, IT'02],

  [Bennatan-Burstein-Caire-Shamai, IT'06],

  [Sun-Liveris-Stankovic-Xiong, ISIT'05].

∗ Vector-perturbation [Peel-Hochwald-Swindlehurst, TCOM'05].

Achievability by "Dirty-Paper Coding" (DPC)

$$X = X_z + X_y$$

$X_z = X_{zc} + X_{zz}$ – as in superposition coding conveys messages (M_c, M_z)

$$E(X_z^2) = (1-\alpha)P$$

X_y – conveys messages (M_y) by DPC against the 'interference' X_z accounting for additive noise σ_y^2.

$$E(X_y^2) = \alpha P \quad \& \quad X_y \perp\!\!\!\perp X_z \,,$$

- Rates:

$$R_c + R_z = \frac{1}{2}\log\left(1 + \frac{(1-\alpha)P}{\alpha P + \sigma_z^2}\right)$$

$$R_y = \frac{1}{2}\log\left(1 + \frac{\alpha P}{\sigma_y^2}\right)$$

* DPC: interference for receiver – y removed @ transmitter.
* receiver y decodes also, in parallel, (R_c, R_z).
* receiver z operates as in superposition coding.

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Entropy Power Inequality

- Converse by EPI [Bergmans, IT'74]
- EPI [Shannon, BSTJ'48], [Stam, IC'59],[Blachman, IT'65]

$$Z^n = X^n + Y^n$$

(X^n, Y^n) independent n-component vectors given U (conditioned version).

$$e^{\frac{2}{n}h(Z^n|U)} \geq e^{\frac{2}{n}h(X^n|U)} + e^{\frac{2}{n}h(Y^n|U)}$$

- Equality $X^n, Y^n|U$ independent Gaussian with proportional covariance matrices

∗ . Proportionality always satisfied for $n = 1$.

Converse by EPI

- Converse by EPI (A. El Gamal, Lecture Notes, EE478)

* $I(U;Z) = h(Z) - h(Z|U)$
* $I(X;Y|U) = h(Y|U) - h(Y|U,X) = h(Y|U) - h(N_y)$
* $Z = X + N_z = X + N_y + N_\Delta = Y + N_\Delta$.

1. $h(Z) \leq \frac{1}{2}\log\left[2\pi e(\sigma_z^2 + P)\right]$, equality $X \sim \mathbb{N}(0,P)$.
2. $\frac{1}{2}\log[2\pi e\sigma_z^2] \leq h(Z|U) \leq h(Z) \leq \frac{1}{2}\log[2\pi e(\sigma_z^2+P)]$
   $\Longrightarrow h(Z|U) = \frac{1}{2}\log[2\pi e(\sigma_z^2+\alpha P)]$, $0 \leq \alpha \leq 1$
3. EPI: $e^{2h(Z|U)} \geq e^{2h(Y|U)} + e^{2h(N_\Delta)}$

$$\Longrightarrow \quad h(Y|U) \leq \frac{1}{2}\log\left(e^{2h(Z|U)} - 2\pi e(\sigma_z^2 - \sigma_y^2)\right)$$
$$= \frac{1}{2}\log[2\pi e(\sigma_y^2 + \alpha P)]$$
$$\Longrightarrow I(U;Z) \leq \frac{1}{2}\log\left(1 + \frac{(1-\alpha)P}{\sigma_z^2+\alpha P}\right)$$
$$\Longrightarrow I(X:Y|U) \leq \frac{1}{2}\log\left(1 + \frac{\alpha P}{\sigma_y^2}\right)$$

- Classics of EPI (conditional version) applications: instrumental in the proof.

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$$\Longrightarrow I(U;Z) \leq \frac{1}{2}\log\left(1 + \frac{(1-\alpha)P}{\sigma_z^2 + \alpha P}\right)$$
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- Classics of EPI (conditional version) applications: instrumental in the proof.

Converse by EPI

- Converse by EPI (A. El Gamal, Lecture Notes, EE478)

* $I(U;Z) = h(Z) - h(Z|U)$
* $I(X;Y|U) = h(Y|U) - h(Y|U,X) = h(Y|U) - h(N_y)$
* $Z = X + N_z = X + N_y + N_\Delta = Y + N_\Delta$.

1. $h(Z) \le \frac{1}{2}\log\left[2\pi e(\sigma_z^2 + P)\right]$, equality $X \sim \mathbb{N}(0,P)$.
2. $\frac{1}{2}\log[2\pi e\sigma_z^2] \le h(Z|U) \le h(Z) \le \frac{1}{2}\log[2\pi e(\sigma_z^2 + P)]$
   $\Longrightarrow h(Z|U) = \frac{1}{2}\log[2\pi e(\sigma_z^2 + \alpha P)]$, $0 \le \alpha \le 1$
3. EPI: $e^{2h(Z|U)} \ge e^{2h(Y|U)} + e^{2h(N_\Delta)}$

$$\Longrightarrow \quad h(Y|U) \le \frac{1}{2}\log\left(e^{2h(Z|U)} - 2\pi e(\sigma_z^2 - \sigma_y^2)\right)$$

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I-MMSE

- The I-MMSE relation [Guo-Shamai-Verdú, IT'05].

$$Y = \sqrt{\text{snr}}\,X + N$$

X	–	Input signal.
Y	–	Output signal.
N	–	Gaussian noise $\sim \mathcal{N}(0,1)$.
snr	–	Signal-to-Noise Ratio.

$$\boxed{\frac{d}{d\text{snr}} I(X;Y) = \frac{1}{2}\text{mmse}(X:\text{snr})}$$

$$\text{mmse}(X:\text{snr}) = E\Big(X - E\big(X|Y\big)\Big)^2.$$

- Generalization: Vectors, continuous time process [Guo-Shamai-Verdú, IT'05].

I-MMSE - EXAMPLES

- I-MMSE: Gaussian Example: $X \sim \mathcal{N}(0,1)$.
  - $\text{mmse}(X_g : \text{snr}) = E\left(X - \frac{\sqrt{\text{snr}}}{1+\text{snr}} Y\right)^2 = \frac{1}{1+\text{snr}}$,
  - $I(X_g; Y) = I_g(\text{snr}) = \frac{1}{2}\log(1+\text{snr})$.
- I-MMSE: Binary Example: $X_b = \pm 1$, symmetric.
  - $\text{mmse}(X_b : \text{snr}) = 1 - \int_{-\infty}^{\infty} \frac{e^{-y^2/2}}{\sqrt{2\pi}} \tanh(\text{snr} - \sqrt{\text{snr}} y)\, dy$
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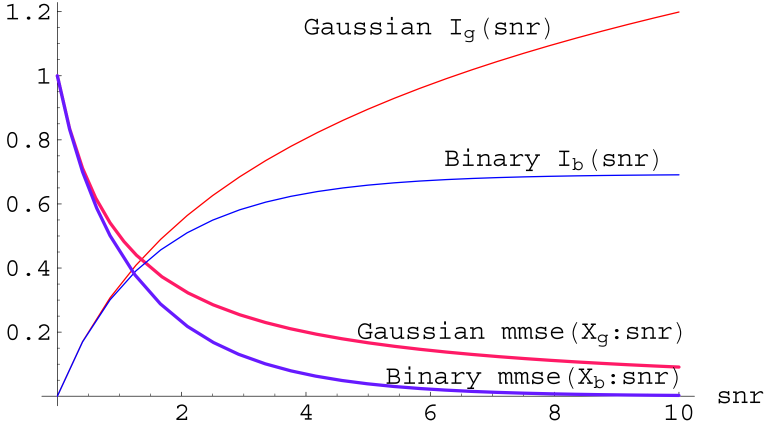
$$\frac{d}{d\text{snr}} I(X;Y) = \frac{1}{2}\text{mmse}(X : \text{snr})$$



- mmse: Unique crossing point

  between MMSEs of a Gaussian and an arbitrary X variable.

  - X be arbitrary zero mean: $E(X^2) = 1$.
  - $X_g \sim \mathcal{N}(0,1)$.
  - $\Delta\text{mmse}(\text{snr}) \stackrel{\triangle}{=} \text{mmse}(\sqrt{\rho}X_g : \text{snr}) - \text{mmse}(X : \text{snr})$

- Given any $\text{snr}_0 > 0$, let $\rho \leq 1$ be the largest number:

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$$\Delta\text{mmse}(\text{snr}) \leq 0\,, \quad \frac{d\Delta\text{mmse}(\text{snr})}{d\text{snr}} \geq 0\,, \quad 0 \leq \text{snr} < \text{snr}_0$$

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  - Arbitrary: $E(X^2) \to b^2\text{mmse}(X : b\,\text{snr}) = \text{mmse}(bX : \text{snr})$.
  - note: mmse $(\sqrt{\rho}X_g : \text{snr}) = \frac{\rho}{1+\rho\text{snr}} \underset{\rho\text{snr} \gg 1}{\sim} \frac{1}{\text{snr}}$.

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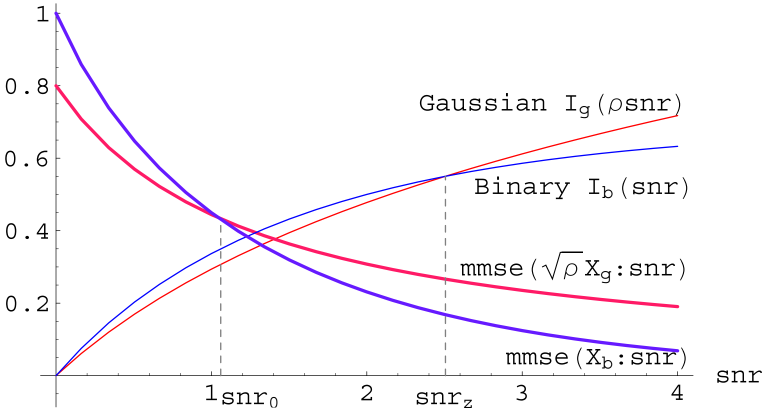
Scaling: $\rho = 0.8$



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- X_u be a zero mean RV dependent on $U = u$.
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Proof on Converse

– Gaussian Broadcast channel

$$Z = \sqrt{\mathrm{snr}_z}\, X + N_z\,,$$
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$$N_y, N_z \sim \mathcal{N}(0,1),\ E(X^2) = 1, \quad \mathrm{snr}_y \geq \mathrm{snr}_z$$

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$$R_y \leq \underline{I(X;Y|U)}$$

$$\bar{R}_z \triangleq R_c + R_z \leq I(U;Z) = I(X,U;Z) - I(X;Z|U)$$
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I-MMSE Expressions

$$I(X;Z|U) = E_U I(X;Z|U=u) = \frac{1}{2}\int_0^{\mathrm{snr}_z} E_U \mathrm{mmse}(X_u : \nu)\, d\nu$$

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$$= \frac{1}{2}\int_0^{\mathrm{snr}_y} E_U \mathrm{mmse}(X_u : \nu)\, d\nu$$
$$= I(X;Z|U) + \int_{\mathrm{snr}_z}^{\mathrm{snr}_y} E_U \mathrm{mmse}(X_u : \nu)\, d\nu$$

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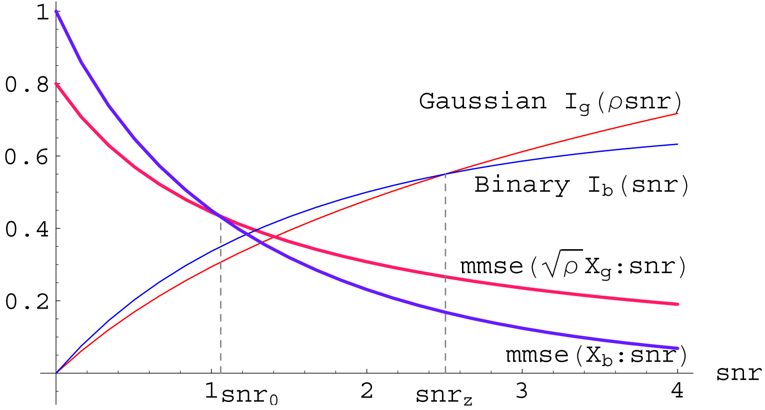
$$I(X;Z|U) = \frac{1}{2}\log(1+\alpha \mathrm{snr}_z) = \frac{1}{2}\int_0^{\mathrm{snr}_z} E_U \mathrm{mmse}(X_u : u)\, d\nu$$

$$= \frac{1}{2}\int_0^{\mathrm{snr}_z} \frac{\alpha}{1+\alpha\nu}\, d\nu$$

$$\frac{d}{d\mathrm{snr}} I(X;Y) = \frac{1}{2}\mathrm{mmse}(X:\mathrm{snr})$$

Scaling: $\alpha = \rho = 0.8$



I-MMSE Expressions

- This implies that:

$$E_U \mathrm{mmse}(X_u; \mathrm{snr}) > \frac{\alpha}{1+\alpha \mathrm{snr}}, \quad 0 \le \mathrm{snr} \le \mathrm{snr}_0 \le \mathrm{snr}_z$$

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- Thus:

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I-MMSE EXPRESSIONS

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- But MMSE is related to entropy [Guo-Shamai-Verdú, IT'05]

$$h(X) = \frac{1}{2}\log(2\pi e) - \frac{1}{2}\int_0^\infty \left\{\frac{1}{1+\nu} - \mathrm{mmse}(X:\nu)\right\} d\nu$$

and can be used elegantly to prove the EPI [Verdú-Guo, IT'06].
- I-MMSE and EPI are related to de Bruijn's identity

$$\frac{\partial h(x+\sqrt{t}N)}{\partial t} = \frac{1}{2}J(X+\sqrt{t}N)$$

- Yet the proof here is based on first principles, addressing only mutual information in a natural way.

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FADING SCALAR BROADCAST CHANNEL

$$\begin{aligned} Z_i &= H_{z,i}X_i + N_{z,i} \\ Y_i &= H_{y,i}X_i + N_{y,i} \end{aligned} , \quad i\text{-time index}$$

- $\{X_i\}$ – power limited input, $E(X^2) = P$.
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- [Biglieri-Proakis-Shamai, IT'98], [Tuninetti-Shamai, DIMACS'04].
- Symmetric fading $H_z \sim H_y \sim H \implies$ degraded BC.
- Gaussian superposition codes $\implies$

$$R_c + R_z \leq E_H \frac{1}{2} \log\left(1 + \frac{|H|^2(1-\alpha)\text{snr}_z}{1+|H|^2\alpha\text{snr}_z}\right), \quad \text{snr}_z = P/\sigma_z^2 ,$$

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Fading Broadcast Channel

- Challenges: Fading BC:
- Degraded Capacity Region:

$$\begin{aligned} R_c + R_z &\leq I(U; Z|H) \\ R_y &\leq I(X; Y|U, H) \end{aligned} \qquad U - X - (Y, H)$$

- Is Gaussian (U, X) optimal as conjectured [Tuninetti-Shamai-Caire, ITA'07] ??
- Problem: Jensen's Penalty in EPI

$$\log\left(e^{E(U)} + 1\right) \leq E \log\left(e^{U} + 1\right).$$

- Partial results:
  - On-Off $(0, 1)$ fading.
  - Finite state fading (uniformly degraded region), $\Longrightarrow$ I-MMSE methodology.
- Other cases:
  - known transmitter CSI [Li-Goldsmith, IT'01]
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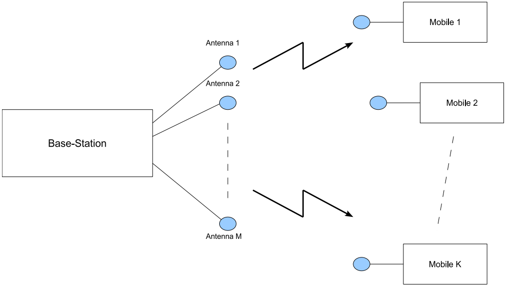
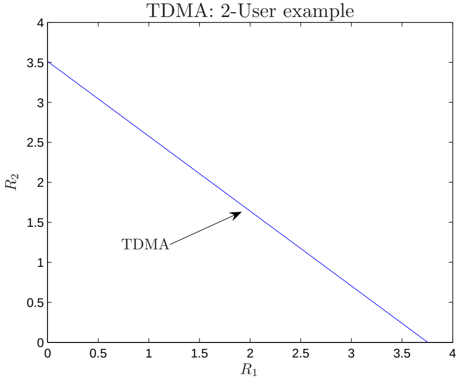
Downlink Channel of a Multi-Antenna Mobile System



$$\mathbf{y}_k = H_k \mathbf{x}_k + \mathbf{n}_k \ , \qquad k = 1 \ldots K$$

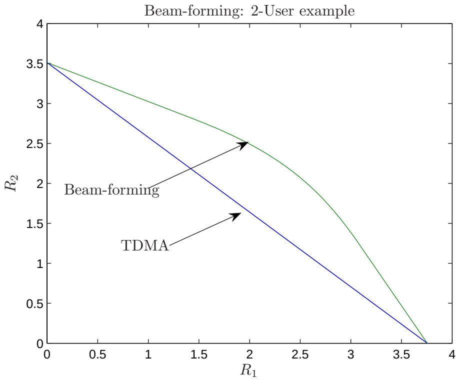
- H_k - Channel fading, $\mathbf{n}_k \sim \mathcal{CN}(\mathbf{0}, \mathbf{N}_k)$- additive noise, $\mathbf{y}_k$ - Received signals.
- Each user receives a different message! $(R_c = 0)$!
- Possible average power constraint: $E(\mathbf{x}^\dagger \mathbf{x}) \leq P$.
- Can we obtain an M-fold increase in throughput?
- In general not a degraded channel!

Time Division Multiple Access



- No multiplicative increase in throughput compared to the single antenna transmitter.

Beam-Forming and Zero-Forcing



- A 2–fold increase in throughput (maximum sum-rate).

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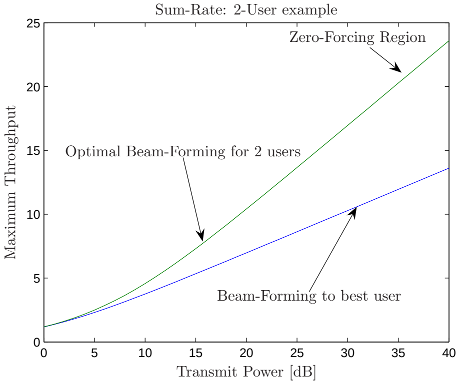

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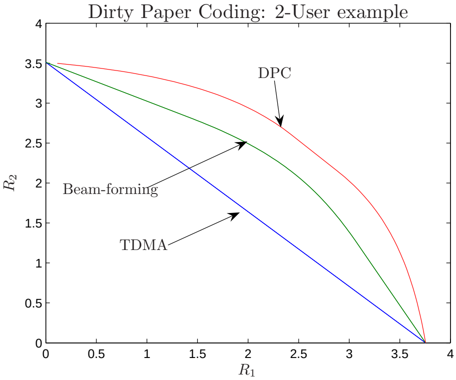

- DPC [Caire-Shamai, IT'03].
- DPC a must not an alternative to superposition!

Historical Perspective

- Non degraded $\Longrightarrow$ open in general.
- The 2–User case $(K = 2)$: [Caire-Shamai, IT'03] sum rate.
  - Costa DPC (achieves Marton's region), [Marton, IT'79].
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MIMO MAC Channel Model: Duality Concepts

- "Reciprocal" MIMO Gaussian MAC:

$$\mathbf{y} = \sum_k \mathbf{H}_k^\dagger \mathbf{x}_k + \mathbf{n}$$

- $\mathbf{n} \sim \mathcal{CN}(\mathbf{0}, \mathbf{N})$.
- Input constraints: individual transmit power, $\mathbb{E}[\mathbf{x}_k^\dagger \mathbf{x}_k] \leq P_k$, total transmit power $\sum_k \mathbb{E}[\mathbf{x}_k^\dagger \mathbf{x}_k] \leq P$.

MIMO MAC: Classical Results

- Capacity region (known from Cover-Wyner):

$$\mathcal{C}_{\text{mac}}(P_1,\ldots,P_K;\mathbf{H}_{1,\ldots,K},\mathbf{N}) = \left\{ \sum_{k\in\mathcal{A}} R_k \leq \log\det\left(\mathbf{I} + \mathbf{N}^{-1}\sum_{k\in\mathcal{A}} \mathbf{H}_k^\dagger P_k \mathbf{H}_k\right),\ \forall\,\mathcal{A}\right\}$$

- Capacity region under sum-power constraint:
  - achieved by Gaussian codes,

$$\mathcal{C}_{\text{mac}}(P;\mathbf{H}_{1,\ldots,K},\mathbf{N}) = \text{c.h.} \bigcup_{\sum_k P_k \leq P} \mathcal{C}_{\text{mac}}(P_1,\ldots,P_K;\mathbf{H}_{1,\ldots,K},\mathbf{N})$$

- Polymatroid structure (Wyner-Cover pentagon): vertices π

$$R_{\pi_k} = \log \frac{\det\left(\mathbf{N} + \sum_{i\leq k} \mathbf{H}_{\pi_i}^\dagger P_{\pi_i} \mathbf{H}_{\pi_i}\right)}{\det\left(\mathbf{N} + \sum_{i<k} \mathbf{H}_{\pi_i}^\dagger P_{\pi_i} \mathbf{H}_{\pi_i}\right)}$$

- Vertices achieved by successive decoding and cancellation: MMSE-DFE interpretation ("Multiuser Detection"). Successive decoding order: $\pi_K, \pi_{K-1}, \ldots, \pi_1$.

MIMO MAC: Classical Results

- Capacity region (known from Cover-Wyner):

$$\mathcal{C}_{\text{mac}}(P_1, \ldots, P_K; \mathbf{H}_{1,\ldots,K}, \mathbf{N}) = \left\{ \sum_{k \in \mathcal{A}} R_k \le \log \det \left(\mathbf{I} + \mathbf{N}^{-1} \sum_{k \in \mathcal{A}} \mathbf{H}_k^\dagger P_k \mathbf{H}_k \right), \forall \mathcal{A} \right\}$$

- Capacity region under sum-power constraint:
  - achieved by Gaussian codes,

$$\mathcal{C}_{\text{mac}}(P; \mathbf{H}_{1,\ldots,K}, \mathbf{N}) = \text{c.h.} \bigcup_{\sum_k P_k \le P} \mathcal{C}_{\text{mac}}(P_1, \ldots, P_K; \mathbf{H}_{1,\ldots,K}, \mathbf{N})$$

- Polymatroid structure (Wyner-Cover pentagon): vertices π

$$R_{\pi_k} = \log \frac{\det\left(\mathbf{N} + \sum_{i \le k} \mathbf{H}_{\pi_i}^\dagger P_{\pi_i} \mathbf{H}_{\pi_i}\right)}{\det\left(\mathbf{N} + \sum_{i < k} \mathbf{H}_{\pi_i}^\dagger P_{\pi_i} \mathbf{H}_{\pi_i}\right)}$$

- Vertices achieved by successive decoding and cancellation: MMSE-DFE interpretation ("Multiuser Detection"). Successive decoding order: $\pi_K, \pi_{K-1}, \ldots, \pi_1$.

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DPC Achievable Region of the MIMO BC

- Let $\mathbf{S} \in \mathbb{S}_+$ be an input covariance constraint. The region

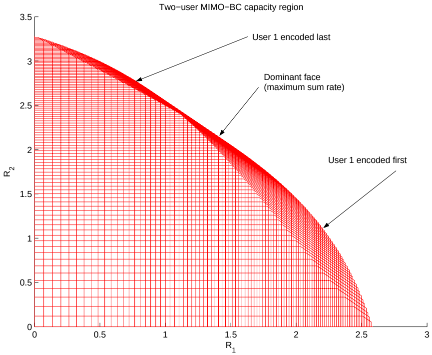
$$\mathcal{R}_{dpc}(\mathbf{S}; \mathbf{H}_{1,\ldots,K}, \mathbf{N}_{1,\ldots,K})$$
$$= \text{c.h.} \bigcup_{\pi} \bigcup_{\sum_k \mathbf{B}_k \le \mathbf{S}} \left\{ \mathbf{R} \,:\, R_{\pi_k} \le \log \frac{\det\left(\mathbf{N}_{\pi_k} + \mathbf{H}_{\pi_k}\left(\sum_{i \le k} \mathbf{B}_{\pi_i}\right)\mathbf{H}_{\pi_k}^\dagger\right)}{\det\left(\mathbf{N}_{\pi_k} + \mathbf{H}_{\pi_k}\left(\sum_{i<k} \mathbf{B}_{\pi_i}\right)\mathbf{H}_{\pi_k}^\dagger\right)} \right\}$$

  is achievable by DPC.

- Achieved by individual Gaussian coding with input covariance matrices $\mathbf{B}_k$. While coding for user π_k, invoke Costa precoding to account all users π_i with $i > k$.
  - Successive precoding order: $\pi_K, \pi_{K-1}, \ldots, \pi_1$.

- $\mathcal{R}_{dpc}(P; \mathbf{H}_{1,\ldots,K}, \mathbf{N}_{1,\ldots,K}) = \bigcup_{\text{tr}(\mathbf{S}) \le P} \mathcal{R}_{dpc}(\mathbf{S}; \mathbf{H}_{1,\ldots,K}, \mathbf{N}_{1,\ldots,K})$.

Duality Concepts



$$\mathcal{R}_{dpc}(P; \mathbf{H}_{1,\dots,K}) = \mathcal{C}_{mac}(P; \mathbf{H}^{\dagger}_{1,\dots,K})$$

- BC region via convex-hull of MAC regions.
- Power allocation and optimal receivers (MMSE-DFE) for the reciprocal MAC are easy to compute.
- General method: solve the dual MAC and map back the solution to the MIMO BC.
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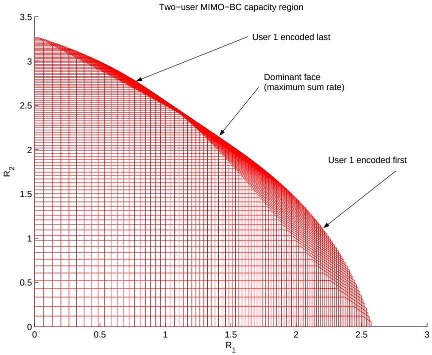

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[WEINGARTEN-STEINBERG-SHAMAI, IT'06]

- Vector EPI $e^{\frac{2}{n}h(X+Y)} \geq e^{\frac{2}{n}h(X)} + e^{\frac{2}{n}h(Y)}$ tight only for (X, Y) Gaussians, with proportional covariances! Why not EPI a la Bergmans?
- Optimality for given covariance constraint $\left(E(XX^\dagger) \preceq S\right)$.
- Optimality for square invertible H_k.
- Aligned MIMO BC – canonic form:

$$\mathbf{y}_k = \mathbf{x} + \mathbf{n}_k \,, \ \mathbf{n}_k \sim \mathcal{CN}(\mathbf{0}, \mathbf{N}_k) \,, \, k = 1, 2 \ldots K \,.$$

- Enhanced Channel: $\mathbf{y}'_k = \mathbf{x} + \mathbf{n}'_k \,, \quad k = 1, \ldots, K$.

  The $\mathbf{y}'_k$ channel is an enhanced version of the $\mathbf{y}_k$ channel if $\mathbf{N}'_k \preceq \mathbf{N}_k \quad \forall k$.
  Clearly, the capacity of the $\{\mathbf{y}'_k\}$ channel is larger than that of the $\{\mathbf{y}_k\}$ channel.

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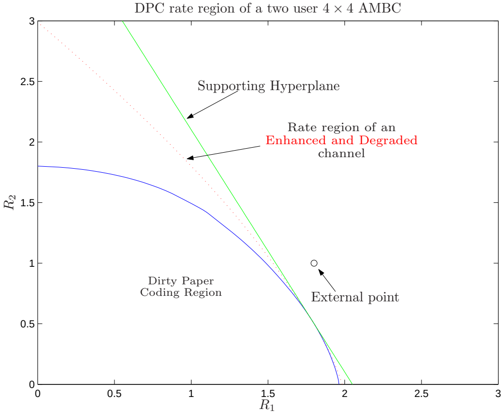
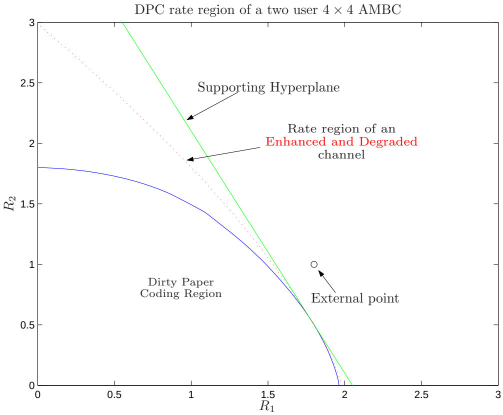
Proof Idea for the Non-Degraded Gaussian Vector Channel



- Step 1: for every point $\mathbf{R} \notin \mathcal{R}_{dpc}(\mathbf{S}; \mathbf{N}_{1,\ldots,K})$, there exists an Enhanced aligned degraded MIMO BC whose DPC region outer bounds the original capacity region and does not contain $\mathbf{R}$.
- Step 2: the capacity region of an Aligned degraded MIMO BC coincides with its DPC region, covariances at the tangential point satisfy equality in vector EPI.

PROOF IDEA FOR THE NON-DEGRADED GAUSSIAN VECTOR CHANNEL



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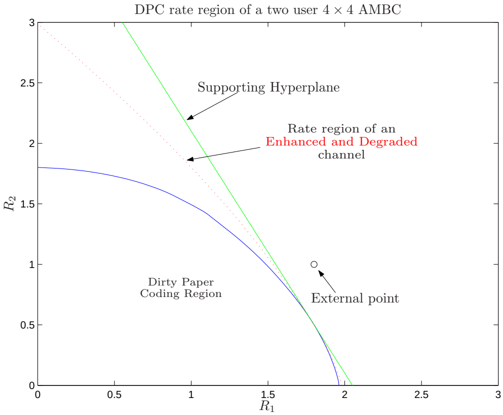

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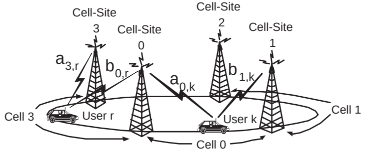
APPLICATION: CELLULAR DOWNLINK – THE WYNER MODEL [SOMEKH-ZAIDEL-SHAMAI, SPWC'05, ARXIV'07]



- A "Wyner-type" multi-cell model with M cells ordered on a *circle*.
- Motivation: symmetry properties, more amenable to analytical analysis, equivalent to linear models for $M \gg 1$.
- A fully synchronous, optimally coded system is assumed, with cell-sites located at the cells' boundaries.
- There are K users in each cell, and a single receive/transmit antenna at each cell-site.
- Each user "sees" only the two nearest cell-sites.
- Models a practical "soft-handoff" scenario at the cells' boundaries.

Downlink System Model

- The received $MK \times 1$ signal vector, is given by

$$\boldsymbol{y}_{dl} = \boldsymbol{H}_M^{\dagger}\boldsymbol{x}_{dl} + \boldsymbol{n}_{dl} \,.$$

  - $\boldsymbol{H}_{M\,[M\times KM]}$ - Channel transfer matrix.
  - $\boldsymbol{x}_{dl\,[M\times 1]}$ - The vector of signals transmitted by the M cell-sites. An equal individual per-cell-site power constraint is assumed: $\left[E\left\{\boldsymbol{x}_{dl}\boldsymbol{x}_{dl}^{\dagger}\right\}\right]_{(m,m)} \leq \bar{P} \quad \forall m.$
  - $\boldsymbol{n}_{dl\,[MK\times 1]} \sim \mathcal{N}_c(\boldsymbol{0}, \boldsymbol{I}_{MK})$ - Circularly symmetric AWGN vector.
- Full CSI is available to the joint multi-cell transmitter only.
- The mobile receivers are assumed to be cognisant of their own CSI, and of the employed transmission strategy.

DOWNLINK AVERAGE PER-CELL SUM-RATE CAPACITY

- Using MIMO-Broadcast-MAC (minmax) duality [Yu, IT'06] the average per-cell sum-rate capacity is:

$$C_{dl}(\bar{P}) = E_{H_M}\left\{\frac{1}{M}\min_{\boldsymbol{\Lambda}_M}\max_{\boldsymbol{\mathcal{D}}_M}\log\frac{\det\left(\boldsymbol{H}_M\boldsymbol{\mathcal{D}}_M\boldsymbol{H}_M^{\dagger}+\boldsymbol{\Lambda}_M\right)}{\det\left(\boldsymbol{\Lambda}_M\right)}\right\}.$$

- The optimization is over all nonnegative diagonal matrices:
  - $\boldsymbol{\mathcal{D}}_M$ $[MK \times MK]$, s.t. $\mathrm{Tr}(\boldsymbol{\mathcal{D}}_M) \leq 1$,
  - $\boldsymbol{\Lambda}_M$ $[M \times M]$, s.t. $\mathrm{Tr}\left(\boldsymbol{\Lambda}_M\right) \leq 1/\bar{P}$.

Downlink - No-Fading

- For non-fading channels $a_{m,k} = b_{m,k} = 1, \forall m, k$.
  - The channel transfer matrix becomes “block-circulant”.
- Average per-cell downlink sum-rate capacity $(M \to \infty)$ is:

$$C_{\text{dl-nf}}(\bar{P}) = \log\left(\frac{1 + 2\bar{P} + \sqrt{1 + 4\bar{P}}}{2}\right) .$$

  - with either average or per cell power constraint and $\forall\, k$.
- Other subsequent results:
  [Foschini-Huang-Karakayali-Valenzuela-Venkatesan, CISS’05],
  [Liang-Goldsmith, GLOBECOM’06],
  [Jing-Tse-Hou-Soriaga-Smee-Padovani, ITA’07].

Cellular Broadcast Channel Models: Challenges

- Fading Models: Bounds in [Somekh-Zaidel-Shamai, arXiv'07].
  - Limiting eigenvalue distribution of finite diagonal $HH^{\dagger}$.
- Planar and general Wyner-like fading models.
- Limited multi-cell processing: cognition, back-haul rate limitations. Partial results in [Somekh-Zaidel-Shamai, arXiv'07], [Lapidoth-Shamai-Wigger, ISIT'07], [Marsch-Fettweis, EW'07], [Sanderovich-Somekh-Shamai, ISIT'07].
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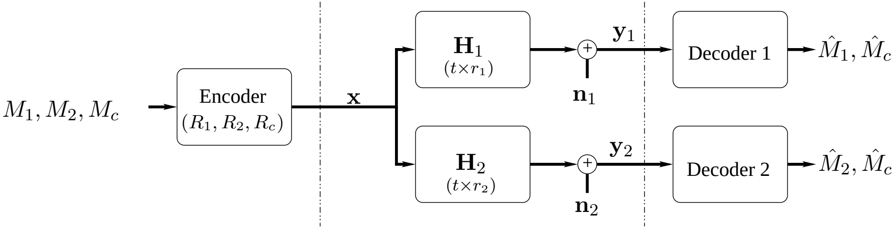
CHALLENGES – COMMON RATE



$$\mathbf{y}_k = \mathbf{H}_k \mathbf{x} + \mathbf{n}_k\,, \quad k = 1, 2 \ldots K \ (K = 2)$$

$$\mathbf{n}_k \sim \mathcal{N}(0, \mathbf{I}), \qquad E\mathbf{x}\mathbf{x}^\dagger \leq \mathbf{S}$$

- What is the *capacity region* ($\mathcal{C}_{\mathrm{C}}(\mathbf{S})$) ?

Achievable Rates – [Jindal-Goldsmith, ISIT'04]

- Allocate powers $\mathbf{Q}_1 + \mathbf{Q}_2 + \mathbf{Q}_c \preceq \mathbf{S} \qquad (K = 2)$.
- $\mathcal{R}^{12}(\mathbf{Q}_1, \mathbf{Q}_2, \mathbf{Q}_c) =$ the set of all (R_1, R_2, R_c) s.t.
  - Common Message - Gaussian coding:

$$R_c \leq \min_{i=1,2} \left\{ \log \frac{|\mathbf{H}_i \mathbf{Q}_c \mathbf{H}_i^T + (\mathbf{H}_i(\mathbf{Q}_1 + \mathbf{Q}_2)\mathbf{H}_i^T + \mathbf{I})|}{|\mathbf{H}_i(\mathbf{Q}_1 + \mathbf{Q}_2)\mathbf{H}_i^T + \mathbf{I}|} \right\}$$

  - Private Message #2 - Gaussian coding and successive cancellation decoding:

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Achievable Rates – [Jindal-Goldsmith, ISIT'04]

$$\mathcal{R}^{12/21}(\mathbf{S}) = \bigcup_{\substack{\mathbf{Q}_1 \succeq 0, \mathbf{Q}_2 \succeq 0, \mathbf{Q}_c \succeq 0 \\ \mathbf{Q}_1 + \mathbf{Q}_2 + \mathbf{Q}_c \preceq \mathbf{S}}} \mathcal{R}^{12/21}(\mathbf{Q}_1, \mathbf{Q}_2, \mathbf{Q}_c)$$

$$\mathcal{R}_{\mathsf{C}} = \textbf{c.h.} \left\{ \mathcal{R}^{12}(\mathbf{S}) \cup \mathcal{R}^{21}(\mathbf{S}) \right\}$$

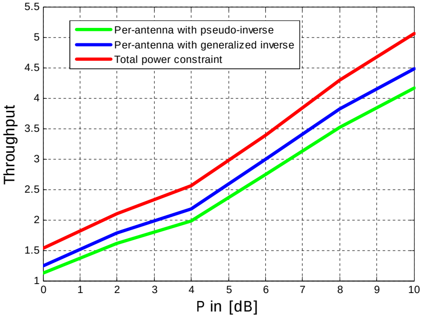
Recent Results! [Weingarten-Steinberg-Shamai, ISIT'06]

- The degraded message set problem $(R_1 = 0$, or $R_2 = 0)$ settled for the multi-antenna broadcast channel with two users.
- Outer bounds suggested and shown to be tight for some parts of the capacity region!
  - * For max sum-rate with a prescribed common rate.
  - * For the aligned channel and for high common rates, $\mathcal{C}_{\mathrm{C}} = \mathcal{R}_{\mathrm{C}}$.
- Challenge: Prove that $\mathcal{C}_{\mathrm{C}} = \mathcal{R}_{\mathrm{C}}$ also for $R_c \leq R_c^{\mathrm{th}}$, demands more than a naive implementation of the enhancement principle.

Challenges: The CMHP-Region - no DPC

- DPC is required to achieve the capacity region of the MIMO Broadcast Channel!
- Suboptimal strategies: beamforming scheduling, linear precoding [Sharif-Hassibi, IT'07]. Nonlinear simplified strategies: vector perturbation & precoding. [Peel-Hochwald-Swindlehurst, COM'05], [Boccardi-Caire, Allerton'05]
- Challenge: What is the optimal region without DPC?
- [Cover, IT'75]; [Van der Meullen, IT'75]; [Hajek-Pursley, IT'79], optimized CMHP region versus [Marton, IT'79].
- Optimized beamforming linear (precoding) – not enough.
- Common rate may play a factor even if not demanded [Amraoui-Kramer-Shamai, ISIT'03].
- Superposition with joint-decoding! not only successive cancelation [Wajcer-Shamai-Wiesel, ITA'06]. Optimization results not necessarily on a vertex MAC point $\Longrightarrow$ joint decoding.
- Transmitter constraints important!

ZERO-FORCING: [WIESEL-ELDAR-SHAMAI, CISS'07]



generalized inverse $\mathbf{H}\mathbf{H}^{gi} = \mathbf{I}$

$$\mathbf{H}^{gi} = \mathbf{H}^{pi} + \left(\mathbf{I} - \mathbf{H}^{pi}\mathbf{H}\right)\mathrm{V}$$

pseudo inverse **(V=0)**

$$\mathbf{H}^{pi} = \mathbf{H}^{\dagger}\left(\mathbf{H}\mathbf{H}^{\dagger}\right)^{-1}$$

We assume $M>K$, and that $\mathbf{H}$ is full row rank

- Zero forcing (pseudo-inverse) with per-antenna power constraint, and optimal linear precoding [Boccardi-Huang, CISS'06, ICASSP'06].
- Fixed receivers oriented linear processing [Wiesel-Eldar-Shamai, TSP'06].
- Pseudo inverse optimal – total power constraint.
- Optimized generalized inverse – per antenna power constraint.

MIMO Gaussian BC: Converse via Extremal -Inequalities

- Alternative converse: Extremal-Inequalities [Liu-Viswanath, IT'07].

$$\max_{\mathbb{P}_X}\left\{h(X+N_1)-\mu h(X+N_2)\right\},\ \mu>1\,.$$

$$\mathbb{P}_X : \mathrm{Cov}(X)\preceq S,\ \mathrm{Cov}(N_1)=K_{N_1},\ \mathrm{Cov}(N_2)=K_{N_2}$$
$\Longrightarrow \mathbb{P}_X$ – Gaussian

- Characterizing the weighted sum-rate

$$\mu_1 R_1+\mu_2 R_2\ ,\qquad \mu_1,\mu_2\geq 0$$

  via the (2–users) Marton-Korner (Theorem 5) [Marton, IT'79] outer bound.

- Challenge: Can this be done naturally and in general, via the standard vector I-MMSE formulism [Guo-Shamai-Verdú] ?

Impact of CSI

$$\begin{aligned} y_i &= (A_i + \tilde{A}_i)\mathbf{x}_i + n_i^y \\ z_i &= (H_i + \tilde{H}_i)\mathbf{x}_i + n_i^z \end{aligned} \qquad i \text{ – time index}$$

2–antenna vector transmitted signal, $\mathbf{x}_i$ is complex and average power constrained:

$$E(|\mathbf{x}|^2) \leq \mathrm{snr}\,.$$

1–antenna scalar receiver signals: y_i, z_i.

Fading (vector) processes $A_i, \tilde{A}_i, H_i \tilde{H}_i$, iid and mutually independent (a simple case),

$$\begin{aligned} E(|A|^2) &= E(|H|^2) = D\,. \\ E(|\tilde{A}|^2) &= E(|\tilde{H}|^2) = \varepsilon\,. \end{aligned}$$

Finite differential entropy proper complex processes: $\tilde{A}_i, \tilde{H}_i$.

n_i^y, n_i^z independent proper scalar AWGN.

CSI: A_i, H_i – available at the transmitter and receivers.
$\tilde{A}_i, \tilde{H}_i$ – available at the receivers only.

Degrees of Freedom

$C_T(\mathrm{snr})$ – throughput (sum-rate).

$DF = \lim_{\mathrm{snr}\to\infty} \frac{C_T(\mathrm{snr})}{\log(\mathrm{snr})}$, degrees of freedom, multiplexing gain.

- Accurate CSI at transmitter and receivers $(\varepsilon = 0)$:
  $DF = 2$ [Caire-Shamai, IT'03].
- MIMO (full cooperation at receivers):
  $DF = 2$ [Telatar, ETT'99], also for $(D = 0)$!
- No CSI at transmitter $(D = 0)$: $DF = 1$ [Caire-Shamai, IT'03].
  Equivalent to a scalar channel [Jafar-Goldsmith, IT'03].
- snr dependent feedback: $\varepsilon \sim \mathrm{snr}^{-1}$: $DF = 2$ [Jindal, IT'06].
- Opportunistic approaches (fixed K): $DF = 0$ [Sharif-Hassibi, IT'05].
- No CSI anywhere: $DF = 0$ (even for MIMO – $\log\log(\mathrm{SNR})$):
  [Lapidoth-Moser, IT'03].

Challenge: D and ε fixed and SNR independent.

$DF =$???

Conjecture: $DF = 1$ (collapse of degrees of freedom).

Equivalent to a MISO (transmission to one user).

- Result: $DF \leq 4/3$ [Lapidoth-Shamai-Wigger, Allerton '05].

Extensions: $A_k, H_k, \tilde{A}_k, \tilde{H}_k$ dependent ergodic processes with memory, and finite conditional (on A_k, H_k) differential entropies of $\tilde{A}_k$ and $\tilde{H}_k$.

Challenge: The Conjecture in Terms of Differential Entropies

Let X and Y be real random variables of variance P. Let U and V be IID zero-mean unit-variance Gaussian random variables such that (U, V) are independent of (X, Y). For any $-\pi \leq \theta < \pi$, let $f^{(\theta)}(\cdot)$ denote the density of

$$(X + U)\cos\theta + (Y + V)\sin\theta$$

and let $h(\theta)$ denote the differential entropy:

$$h(\theta) = -\int_{-\pi}^{\pi} f^{(\theta)}(\xi)\log f^{(\theta)}(\xi)\, d\xi \,.$$

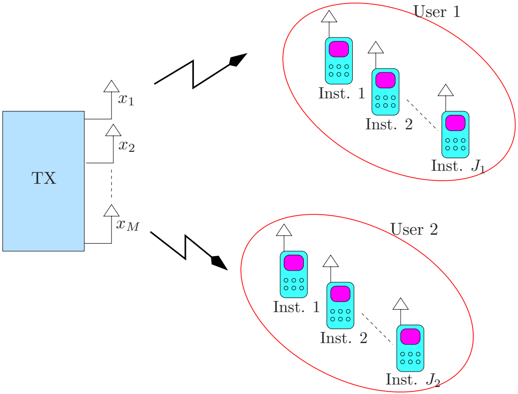
$$h_{\text{sup}} \triangleq \sup_{-\pi\leq\theta<\pi} h(\theta) \,.$$

Let h_{avg} denote the average of $h(\theta)$ w.r.t. a **fixed bounded** density $f_\Theta(\theta)$:

$$h_{\text{avg}} = \int_{-\pi}^{\pi} f_\Theta(\theta) h(\theta)\, \mathrm{d}\theta \,.$$

$$\sup_{P>0} \sup_{X,Y,\text{s.t.}:E[X^2],E[Y^2]<P} \{h_{\text{sup}} - h_{\text{avg}}\} \stackrel{?}{<} \infty \,.$$

Compound BC



$y_k^j = \mathbf{H}_k^j \mathbf{x} + n_k^j$

$n_k^j \sim \mathcal{NC}(0,1)$

$k = 1, \ldots, K$ (groups)

special case: $K = 2$

$j = 1, \ldots, J_k$

(instances/

users per group)

or Groups of users with common messages.

Compound BC – Related Results

- Degraded compound broadcast channels
  - Parallel channels [Diggavi-Tse, ITW'06].
  - MIMO-Broadcast [Weingarten-Liu-Shamai-Steinberg-Viswanath, ISIT'07].
  - Strict degradation order: 'channel 1 better than 2 for any possible realization'.

$$\Longrightarrow \quad R_1 = \min_{j=1,\ldots J_1} \log \det(I + \mathbf{H}_1^j Q \mathbf{H}_1^{j\dagger})$$

$$R_2 = \min_{j=1,\ldots J_2} \log \frac{\det(I+\mathbf{H}_2^j S \mathbf{H}_2^{j\dagger})}{\det(I+\mathbf{H}_2^j Q \mathbf{H}_2^{j\dagger})}$$

for some covariance Q under the constraint $\mathrm{Cov}(\mathbf{x}) \preceq S$.

- Multiplexing gain region [Weingarten-Kramer-Shamai, ITA'07].
- Scaling laws (large number of users, antennas) in MIMO group-broadcast channels [Dana-Al Naffouri-Hassibi, ISIT'07].

Multiplexing Gain Region

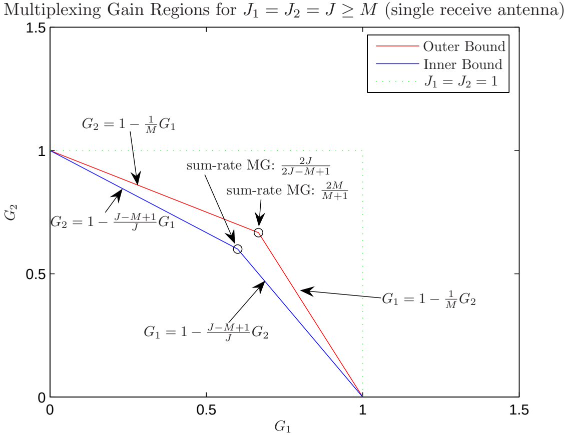
Definition

The multiplexing gain region is the set of all achievable limit points

$$\lim_{\mathrm{snr}\to\infty}\left(\frac{R_1(\mathrm{snr})}{\log \mathrm{snr}}, \frac{R_2(\mathrm{snr})}{\log \mathrm{snr}}\right) = (G_1, G_2)$$

- The multiplexing gain region is always convex.

EXAMPLE: [WEINGARTEN-KRAMER-SHAMAI, ITA'07]



- Tight for $J = M$!

CHALLENGES: [WEINGARTEN-KRAMER-SHAMAI, ITA'07]

CONJECTURE

If any set of M vectors out of $\mathbf{H}_1^1, \mathbf{H}_1^2, \ldots, \mathbf{H}_1^{J_1}, \mathbf{H}_2^1, \mathbf{H}_2^2, \ldots, \mathbf{H}_2^{J_2}$ are linearly independent, the multiplexing gain region is given by

$$G_1 \leq 1 - \frac{\max(0, J_1 - M + 1)}{J_1} G_2,$$
$$G_2 \leq 1 - \frac{\max(0, J_2 - M + 1)}{J_2} G_1.$$

- Determine the rate region of the compound MIMO broadcast channel, with no specific degradation order.

CHALLENGES: [WEINGARTEN-KRAMER-SHAMAI, ITA'07]

CONJECTURE

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Multiuser Scaling & Opportunistic Approaches

- Multiuser Scaling & Opportunistic Approaches
  - Optimal scaling (fixed snr) $\sim M \log(N \log K)$ M-transmit antennas, K-users, N-receive antenna's per user. [Xie-Georghiades, TWC'06].
  - Opportunistic random-beamforming and related strategies [Viswanath-Tse-Laroia, IT'02], [Sharif-Hassibi, IT'05], [Baesteh-Khandani, arXiv'07].
- Scheduling in multiuser regimes: dual-MAC [Yu-Ree, TCOM'06] simultaneous transmission to no more than M^2 users (no more than N^2 beams per user).

- snr scaling as to achieve full sum-rate

$$R_s \sim M \log(\mathrm{snr}) \,, \quad \mathrm{snr} \gg 1$$

within $\Delta R = \log_2 b$.

$\longrightarrow$ *ZF* requires accuracy in CSI estimation [Jindal, IT'06] proportional to $\left(\frac{\mathrm{snr}}{b-1}\right)^{-1}$.

$\Longrightarrow$ feedback rate: $(M-1)\log\Big(\mathrm{snr}/(b-1)\Big)$.

- Mandatory scaling for arbitrary processing (under certain assumptions) [Caire-Jindal-Shamai, Asilomar'07].
- Efficient feedback schemes accounting for receiver inaccuracies [Caire-Jindal-Kobayashi-Ravindran, ISIT'07].

Challenges: MIMO GBC-CSI

- Optimal (not necessarily ZF based!) non asymptotic VGBC approach in the realm of imprecise CSI @ transmitter: rate region + common rate!

∗ Does optimal processing relate to 'writing on fading paper'?
$Y = H(X + S) + N$, H not fully known @ transmitter,
S interference known @ transmitter un-causally
[Bennatan-Burshtein, Allerton'06].

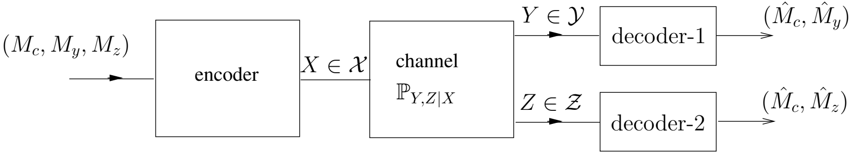
∗ If so, under which conditions are Costa's linear relations
$U = FX + BS$ (F, B matrices, X, S independent) optimal?

∗ Common rate (included) $\Longrightarrow$ relations to 'Carbon Copy'!
[Khisti-Erez-Lapidoth-Wornell, IT'07].

∗ Common rate only – standard compound setting
[Wiesel-Eldar-Shamai, TWC'07].

HISTORICAL PERSPECTIVE

T. M. Cover, "Broadcast Channels," IEEE Trans. Inform. Theory, vol. IT–18, no. 1, pp. 2–14, January 1972.



(M_c, M_y, M_z) common/separate messages.

$X \in \mathcal{X}$ channel input: subjected to input constraints, e.g. $E(X^2) \leq P$.

$Y \in \mathcal{Y}$, $Z \in \mathcal{Z}$ – channel outputs.

Marton's Achievable Rate Region

- [Marton, IT'79] (R_c, R_y, R_z) – achievable (Marton Region).

$$
\begin{aligned}
R_c &\leq \min\Big\{I(W;Y), I(W;Z)\Big\} \\
R_c + R_y &\leq I(W,U;Y) \\
R_c + R_z &\leq I(W,V;Z) \\
R_c + R_y + R_z &\leq \min\Big\{I(W;Y), I(W;Z)\Big\} + I(U;Y|W) + I(V;Z|W) - I(U;V|W)
\end{aligned}
$$

$$\mathbb{P}_{W,V,U,X,Y,Z} = \mathbb{P}_{WUV}\mathbb{P}_{X|WUV}\mathbb{P}_{YZ|X}\,.$$

∗ [Gelfand-Pinsker, PPI'80] – mentions R_c explicitly.

- Tight $\Longrightarrow$ all special cases mentioned + (MIMO-GBC).
- Coding idea: binning $\Longrightarrow$ auxiliary $rv(U, V, W)$.
- GP is a vertex point
  $\{R_c, R_y, R_z\} = \{\min[I(W;Y), I(W;Z)], I(U;Y|W), I(V;Z|W) - I(V;U|W)\}$

∗ special case $\mathbb{P}_{W,V,U} = \mathbb{P}_W\mathbb{P}_V\mathbb{P}_U$

$\Longrightarrow$ CMHP region: [Cover, IT'75; Van der Meullen, IT'75; Hajek-Pursley, IT'79].

OUTER REGION

- [Nair-El Gamal, IT'07]

  (R_c, R_y, R_z)

  $R_c < \min\Big\{I(W;Y), I(W;Z)\Big\}$

  $$\begin{array}{lcl} R_c + R_y & \leq & I(W,U;Y) \\ R_c + R_z & \leq & I(W,V;Z) \\ R_c + R_y + R_z & \leq & I(W,U;Y) + I(V;Z|U,W) \\ R_c + R_y + R_z & \leq & I(W,V;Z) + I(U;Y|V,W)\,. \end{array}$$

  for some $\mathbb{P}_U\mathbb{P}_V\mathbb{P}_{X|U,V}\mathbb{P}_{Y,Z|X}$.

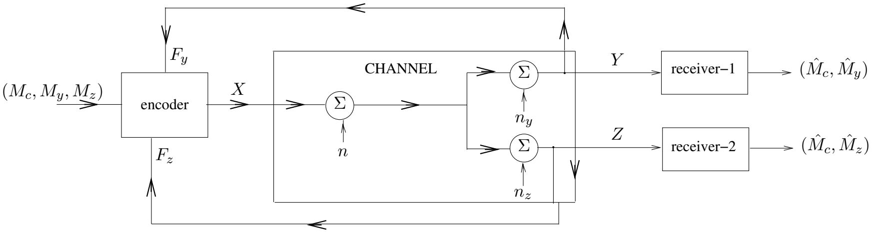
- [Korner-Marton, Theorem 5, IT'79]

  (R_y, R_z)

  $$\begin{array}{lcl} R_y & \leq & I(X;Y) \\ R_z & \leq & I(V;Z) \\ R_y + R_z & \leq & I(X;Y|V) + I(V;Z) \end{array}$$

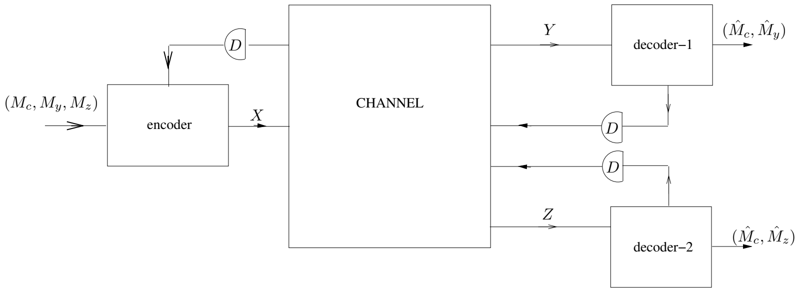
- [Korner-Marton, IT'79] $\Longrightarrow$ enhanced region.

Broadcast Channels: Noiseless Feedback



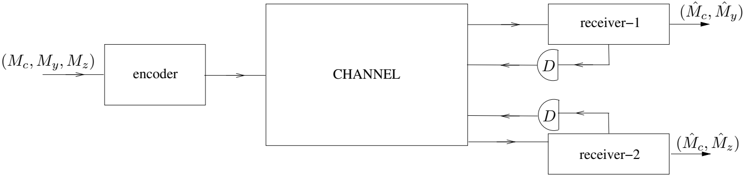
- Capacity is not increased by feedback for physically degraded channels [El-Gammal, IT'78].
- Capacity is increased by double-sided feedback in a Gaussian (stochastically degraded) channel [Ozarow-Leung-Yan-Chong, IT'84].
- Capacity may increase even with one-sided feedback [Bhaskaran, ISIT'07].

A Network Oriented Outlook



- A generalized feedback model, accounts for
  - Shannon feedback.
  - Receiver cooperation.
  - Relaying.

Broadcast Channels: Receiver Cooperation & Relay



Cooperation:

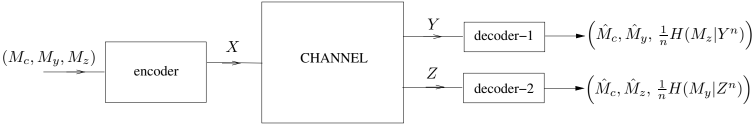
- Bounds and capacity regions in certain degraded cases [Liang-Veeravalli, IT'07].
- Multi hop receiver (orthogonal) cooperation, bounds and capacity in certain degraded cases [Dabora-Servetto, IT'06].

Relaying:

- Inner and outer bounds for this general model + capacity region in certain cases [Liang-Kramer, IT'07], [Bhaskaran, EPFL'07].
- Iterative decoding of a broadcast (common) message [Draper-Frey-Kshischang, Allerton'03].
- One-shot conferencing [Ng-Maric-Goldsmith-Shamai-Yates, ITW'06].
- Broadcast cooperating strategies [Steiner-Sanderovich-Shamai, IT'07].

∗ Unified View [Kramer-Maric-Yates, FnT'07].

SECRECY



- Conditional entropy measures ‘Shannon wise’ confidentiality.
- Broadcast channel with confidential message [Csiszar-Korner, IT’78].
- Two confidential messages [Liu-Maric-Spasojevic-Yates, Allerton ’06].
- Wireless fading channels [Gopala-Lai-El Gamal, IT’07].
- Independent parallel channels [Li-Yates-Trappe, Allerton’06].
- Fading and parallel channels [Liang-Poor-Shamai, ISIT’07].

THE BROADCAST APPROACH

- In static compound\composite channels the different possible realizations are treated as different receivers within a broadcast channel framework [Cover, IT'72].
- Fading scalar channels [Shamai, ISIT'97].
- MIMO models [Shamai-Steiner, IT'03].
- Multiple access fading channels [Shamai, ISIT'00], [Minero-Tse, ISIT'07].
- Partial state knowledge @ transmitter [Steiner-Shamai, TWC'07].
- Two-Hop relay fading channels [Steiner-Shamai, IT'06].
- Broadcast cooperation strategies in broadcast channels [Steiner-Sanderovich-Shamai, IT'07].

SOURCE-CHANNEL, DISTORTION, SUCCESSIVE REFINEMENT & BROADCASTING

- The target is to adapt achievable distortion, rather than rate, to the channel state available @ the receiver end only.

∗ Marriage between successive refinement [Rimoldi, IT'04], and broadcast approach [Shamai, ISIT'97].

∗ Distortion exponents [Caire-Narayanan, Allerton'05], [Gunduz-Erkip, Asilomar'05], [Bhattad-Narayanan-Caire, arXiv'07].

∗ Recursive algorithms-expected distortion [Ng-Gunduz-Goldsmith-Erkip, ISIT'07, ICC'07].

∗ Variational approach (continuous case) + efficient recursive algorithms [Tian-Steiner-Shamai-Diggavi, ITW'07].

JOINT SOURCE-CHANNEL CODING

- Distortion region for transmitting source $\{S\}$ over a broadcast channel $\mathbb{P}_{y_1,y_2,\ldots,y_k}|X$.
  - no source channel separation in general.
- Gaussian BC: $Y_k = \sqrt{\text{snr}_k}X + N\,, \quad k = 1, 2 \ldots K$.
  Gaussian source $\{S\}$ same bandwidth B (Bandwidth expansion) =1, $S = X$ optimal!
  Analogue is not just an alternative to digital, as in a single user case: code - or not code [Gatspar-Rimoldi-Vetterli, IT'03], but in fact is a must!
- Back to (some) analogue? general B.
  - [Mittal-Phamdo, IT'02].
  - [Reznic-Feder-Zamir, IT'06]: efficient region for $B > 1$.
  - [Caire-Narayanan, Allerton'05]: efficient region for $B < 1$.
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- Challenge: distortion region over the (Gaussian) source-BC (general B).

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Side Information @ Receivers

⟹ Coding with different degrees of SI (motivated by: analog signals) @ broadcast receivers.

- Broadcast interactive Wyner-Ziv and Slepian-Wolf setting.

  [Heegard-Berger, IT'85], [Kaspi, IT'94], [Steinberg-Merhav, IT'04]

  [Tian-Diggavi, ITA'06], [Wolf, CISS'04], [Tuncel, IT'06]

  [Ng-Tian-Goldsmith-Shamai, ITW'07].

- Challenges:

  Distortion region (not only exponents) on Gaussian source over Gaussian broadcast channel with bandwidth expansion $(B > 1)$, and analogue component (SI).

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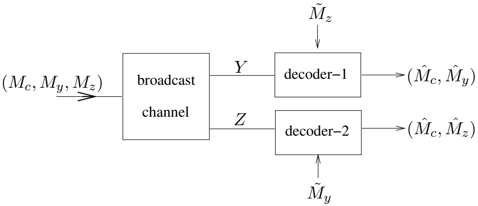

- Applications: Back-relaying $\Longrightarrow$ Two-Way Relaying.
- Network coding (butterfly: $X = M_y \oplus M_z, \tilde{M}_z = M_z, \tilde{M}_y = M_y$) [Rankov-Wittneben, ISIT'06], [Xie, CTW'07], [Oechtering-Schnurr-Bjelakovic-Boche, CISS'07].
- **Challenges:** Achievable and Outer bounds on broadcast channels with general SI.

  - Under which conditions these bound meet: capacity region with SI not necessarily a harder problem.

  * **Recent results in:** [Kramer-Shamai, ITW'07].

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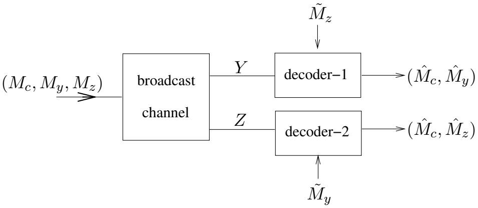

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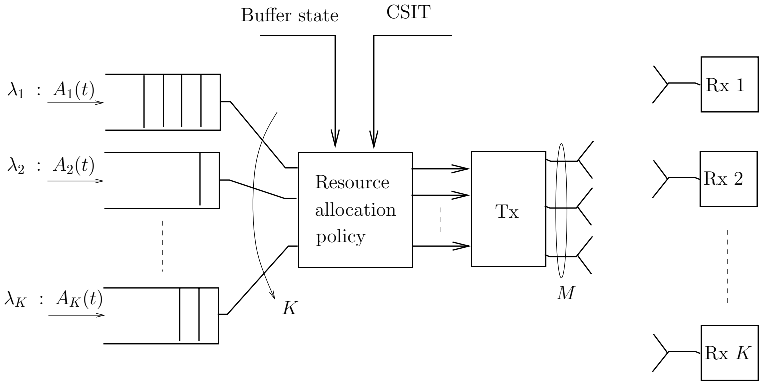
Network Related Broadcast Problems

- Queues & Broadcast Channels.



Network Related Broadcast Problems

- Stability & Scheduling

$*$ stability region $\{\lambda\}$, for which resource allocation policy stabilizes queues: $\{\lambda\} \equiv C$ (ergodic capacity).

$\Longrightarrow$ optimization of weighted (queue-dependent) rates. [Neely-Modiano-Rohrs, ATNet'03], [Yeh-Cohen, ISIT'04] [Boche-Wiczanowski, WC'06].

Network Related Broadcast Problems

- Broadcast approach & queues [Steiner-Shamai, CISS'05] & (ARQ) [Steiner-Shamai, TWC'07].
- Delay-Limited Broadcast channel capacity.
  Resource (power, bandwidth, scheduling) allocation given rate demands. [Li-Goldsmith, IT'01], [Jindal, ISIT'06], [Kobayashi-Caire, JSAC'06], [Seong-Narashimhan-Cioffi, JSAC'06], [Schubert-Boche, FnT'05], [Mohseni-Chang-Cioffi, JSAC'06], [Michel-Wunder, ISIT'07].
- Challenges: General QoS & rate demands.

Network Related Broadcast Problems

- Correlated sources over broadcast channels [Han-Costa, IT'87], [Choi-Pradhan, CISS'05].
- Streaming broadcasting [Cover, IT'98], [Shulman-Feder, ISIT'00, ITW'02].
  - ∗ Fountain capacity a la [Shamai-Telatar-Verdú, ISIT'06] ?
- State-dependent broadcast channels
  [Gelfand-Pinsker, ITS (Tashkent) '84]
  [Steinberg, IT'05], [Steinberg-Shamai, ISIT'05]
  [Sigurjonsson-Kim, ISIT'05].
  - ∗ capacity region ?
- Broadcast channels in the wideband regime: first (power) and second (slope) order optimality [Lapidoth-Telatar-Urbanke, IT'03], [Caire-Tuninetti-Verdú, IT'04].
- Error Exponents with/without feedback? [Haroutunian-Haroutunian-Harutyunyan, FnT'07].

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  [Cover, IT'98], [Shulman-Feder, ISIT'00, ITW'02].
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- State-dependent broadcast channels
  [Gelfand-Pinsker, ITS (Tashkent) '84]
  [Steinberg, IT'05], [Steinberg-Shamai, ISIT'05]
  [Sigurjonsson-Kim, ISIT'05].
  - ∗ capacity region ?
- Broadcast channels in the wideband regime: first (power) and second (slope) order optimality [Lapidoth-Telatar-Urbanke, IT'03], [Caire-Tuninetti-Verdú, IT'04].
- Error Exponents with/without feedback?
  [Haroutunian-Haroutunian-Harutyunyan, FnT'07].

NETWORK RELATED BROADCAST PROBLEMS

- Correlated sources over broadcast channels
  [Han-Costa, IT'87], [Choi-Pradhan, CISS'05].
- Streaming broadcasting
  [Cover, IT'98], [Shulman-Feder, ISIT'00, ITW'02].
  - ∗ Fountain capacity a la [Shamai-Telatar-Verdú, ISIT'06] ?
- State-dependent broadcast channels
  [Gelfand-Pinsker, ITS (Tashkent) '84]
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CONCLUDING COMMENTS

- The broadcast channel, in its general interpretation, is now a central building block in modern communication networks.

$\Longrightarrow$ Motivates a plethora of challenging theoretical problems.

$\Longrightarrow$ Interesting theoretical results and techniques inspire approaches in practical systems, i.e. linear/nonlinear precoding.

- Is it some inspiration and a 'new look' that we need to settle the longstanding problems of the full capacity region? Or is it basic new tools that we lack (binning is not enough!) and neither are simple manipulations of Fano's inequality?

$\ast$ Do classical single letter expressions capture the general broadcast channel setting?

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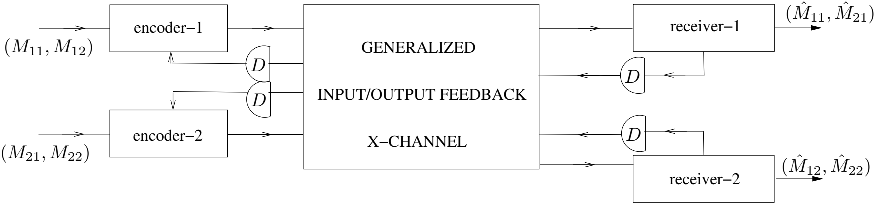
X-CHANNEL WITH GENERALIZED INPUT/OUTPUT FEEDBACK

- Generalizations motivated by a network perspective:

  The X-channel:
  – (encompasses: broadcast, interference and multiple access channels).



  Recent results [Maddah-Ali-Motahari-Khandani, ISIT'06], [Devroy-Sherif, ISIT'07], [Jafar-Shamai, arXiv'06] demonstrate interesting features of multi-antenna X-channels beyond the special cases of multiple access, broadcast and interference channels with/without cognitive information @ transmitters even in terms of degrees of freedom.

LITERATURE & TUTORIALS

- Literature

Apology: For not mentioning many dozens of relevant studies, for the interest of time and space, and limited familiarity.

- Tutorials

1. T.M. Cover, "Comments on the Broadcast Channel," IEEE Trans. Inform. Theory, vol. 44, no. 6, pp. 2524-2530, Oct. 1998.
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