Sum-Rates for Wyner-type C-RAN Uplink with Inter-Connected Oblivious Radio Units

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Outline

- Introduction
- System Model
- Point-to-Point Compression
- Leveraging Side Information
- Joint Decompression and Decoding
- Concluding Remarks
Base Stations (BSs) operate as radio units (RUs) [China][Simeone et al:JCN].

Baseband processing takes place in the “cloud”.

Fronthaul links carry complex (IQ) baseband signals [CPRI][IDC][Andrews et al:JSAC].

Advantages

- Low-cost deployment of BSs [China][Segel-Weldon]
- Effective interference mitigation via joint baseband processing [Shamai et al:JWCC][Somekh et al:TIT]

State-of-the-Art
Distributed fronthaul compression


\[ \pi \] denotes a permutation of RUs’ indexes.

\[ \hat{y}_{\pi(1)}^{ul} \]

\[ \hat{y}_{\pi(2)}^{ul} \]

\[ \hat{y}_{\pi(N_R)}^{ul} \]
Joint decompression and decoding (JDD)

Inter-RU cooperation for non-cooperative cellular systems:

- Analysis for Wyner, Circular Wyner models
  [Simeone et al:TIT][Simeone et al:FnT]

- Other UE and/or Cell-Sites cooperation in Wyner Model [Wigger et al:TIT]
Main Contributions

- Inter-RU cooperation for the uplink of C-RAN:
  - Analysis for circular Wyner model
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**System Model**

- Wyner-type C-RAN uplink
  - $N$ pairs of RU-UE ($\mathcal{N} = \{1, 2, \ldots, N\}$)
  
- Fronthaul connections
  - $C$ bit/symbol between RU-CU
  - $B$ bit/symbol between RU-RU

- Uplink channel

\[
Y_i = X_i + \alpha X_{[i-1]} + Z_i,
\]

where
- $Y_i$: Rx signal RU $i$,
- $X_i$: Tx signal of UE $i$,
- $Z_i$: Noise at RU $i$ with $Z_i \sim N(0, \sigma^2)$,
- $\alpha$: Inter-cell channel gain with $\alpha \in [0,1]$.

<Example for $N = 3$>
Encoding at UEs

- Encoding at UE $i$
  - Message $M_i \in \{1, 2, \ldots, 2^{nR_i}\}$
    
    where $R_i$ is the rate of the message,
    
    $n$ is the coding block length (assumed to be sufficiently large).

- Encoded signal $X_i$
  - Message $M_i$ is encoded to obtain an encoded signal
    
    $$X_i \sim N(0, P).$$

- Signal-to-noise ratio (SNR) of the uplink channel
  
  $$\text{SNR} = \frac{P}{\sigma^2}.$$
Oblivious Processing at RUs

- In-network processing (INP) at RU \( i \)

- Oblivious/Nomadic: no structure information (code-books) of UE's is available at the RUs
In-network processing (INP) at RU $i$

Oblivious/Nomadic: no structure information (code-books) of UE's is available at the RUs
In-network processing (INP) at RU $i$

$\hat{Y}_{B,i} = Y_i + Q_{B,i}$, with quantization noise $Q_{B,i} \sim N(0, \omega_{B,i})$

- Oblivious/Nomadic: no structure information (code-books) of UE's is available at the RUs
Oblivious Processing at RUs

- In-network processing (INP) at RU $i$

- Oblivious/Nomadic: no structure information (code-books) of UE's is available at the RUs

\[ S_i = \gamma_i \hat{Y}_{B,[i-1]} + Y_i \]

(Linear is optimal.)
In-network processing (INP) at RU \( i \)

\[
\hat{Y}_{C,i} = S_i + Q_{C,i}
\]

with \( Q_{C,i} \sim N(0, \omega_{C,i}) \)

\[
S_i = \gamma_i \hat{Y}_{B,[i-1]} + Y_i
\]

(Linear is optimal.)

Oblivious/Nomadic: no structure information (code-books) of UE's is available at the RUs
Decoding at CU

- Decompression and decoding at CU
  - CU recovers the quantized INP output signals \( \hat{Y}_{C,1}, \hat{Y}_{C,2}, \ldots, \hat{Y}_{C,N} \).
Decoding at CU

- Decompression and decoding at CU
  - CU recovers the quantized INP output signals $\hat{Y}_{C,1}, \hat{Y}_{C,2}, \ldots, \hat{Y}_{C,N}$.
Decompression and decoding at CU

- CU recovers the quantized INP output signals $\hat{Y}_{C,1}, \hat{Y}_{C,2}, \ldots, \hat{Y}_{C,N}$.
- Then, it jointly decodes the messages $\hat{M}_1, \hat{M}_2, \ldots, \hat{M}_N$. 

![Diagram of Decoding at CU](image-url)
Decompression and decoding at CU

- CU recovers the quantized INP output signals $\hat{Y}_{C,1}, \hat{Y}_{C,2}, \ldots, \hat{Y}_{C,N}$.
- Then, it jointly decodes the messages $\hat{M}_1, \hat{M}_2, \ldots, \hat{M}_N$.

$$R_{\text{sum}} = \sum_{i \in \mathcal{N}} R_i = I(\{X_i\}_{i \in \mathcal{N}}; \{\hat{Y}_{C,i}\}_{i \in \mathcal{N}})$$
Decoding at CU

- Vector expression of quantized signals $\{\hat{Y}_{C,i}\}_{i \in \mathcal{N}}$

\[
\begin{bmatrix}
\hat{Y}_{C,1} \\
\hat{Y}_{C,2} \\
\vdots \\
\hat{Y}_{C,N} \\
\hat{Y}_C
\end{bmatrix}
= 
\begin{bmatrix}
X_1 \\
X_2 \\
\vdots \\
X_N
\end{bmatrix}
+ 
\begin{bmatrix}
Z_1 \\
Z_2 \\
\vdots \\
Z_N
\end{bmatrix}
+ 
\begin{bmatrix}
Q_{B,1} \\
Q_{B,2} \\
\vdots \\
Q_{B,N}
\end{bmatrix}
+ 
\begin{bmatrix}
Q_{C,1} \\
Q_{C,2} \\
\vdots \\
Q_{C,N}
\end{bmatrix},
\]

where $H_X = I + (\gamma + \alpha)E_1 + \gamma \alpha E_2$, with $E_1$ = circulant matrix with first row $[0 \cdots 0 0 1]$, $E_2$ = circulant matrix with first row $[0 \cdots 0 1 0]$.

$H_Z = I + \gamma E_1$,

$H_Q = \gamma E_1$,

(We have $E_1E_1^T = E_2E_2^T = I$, $E_1E_2^T = E_1^T$, $E_2E_1^T = E_1$)
Decoding at CU

- Sum-rate $R_{\text{sum}}$ can be written as

$$R_{\text{sum}} = I \left( \{ X_i \}_{i \in \mathcal{N}} ; \{ \hat{Y}_{C,i} \}_{i \in \mathcal{N}} \right)$$

$$= \frac{1}{2} \log_2 \det \left( I + P \left( \sigma^2 H_Z H_Z^T + \omega_B H_Q H_Q^T + \omega_C I \right)^{-1} H_X H_X^T \right)$$

$$= \frac{1}{2} \sum_{i \in \mathcal{N}} \log_2 \left( 1 + P \frac{1 + (\gamma + \alpha)^2 + \gamma^2 \alpha^2 + (\gamma + \alpha)(1 + \gamma \alpha) \lambda_{1,i} + \gamma \alpha \lambda_{2,i}}{\sigma^2 \gamma \lambda_{1,i} + \sigma^2 (\gamma^2 + 1) + \omega_B \gamma^2 + \omega_C} \right),$$

where $\lambda_{k,l}$: lth largest eigenvalue of $E_k + E_k^T$ given as

$$\lambda_{k,l} = 2 \cos \left( 2k \pi \frac{l - 1}{N} \right).$$
Design Space

- **Optimization variables**
  - $\omega_B$: quantization noise power for RU-RU links
  - $\omega_C$: quantization noise power for RU-CU links
  - $\gamma$: combining coefficient for in-network processing

- **Objective function**
  - Sum-rate $R_{\text{sum}}$

- **Constraints**
  - Capacity $B$ of RU-RU links
  - Capacity $C$ of RU-CU links
Design Space

- **Optimization variables**
  - $\omega_B$ : quantization noise power for RU-RU links
  - $\omega_C$ : quantization noise power for RU-CU links
  - $\gamma$ : combining coefficient for in-network processing

- **Objective function**
  - Sum-rate $R_{\text{sum}}$

- **Constraints**
  - Capacity $B$ of RU-RU links
  - Capacity $C$ of RU-CU links

Modeled differently depending on decompression strategy.
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In this strategy, the quantized signals $\hat{Y}_{B,i}$ and $\hat{Y}_{C,i}$ are decompressed without leveraging side information.

**Constraints on $\omega_B$ for RU-RU links** [ElGamal-Kim, Ch. 3]

$$I(Y_i;\hat{Y}_{B,i}) = \frac{1}{2} \log_2 \left(1 + \frac{P(1+\alpha^2)+\sigma^2}{\omega_B}\right) \leq B.$$ 

**Constraints on $\omega_C$ for RU-CU links** [ElGamal-Kim, Ch. 3]

$$I(\gamma\hat{Y}_{B,[i-1]} + Y_i;\hat{Y}_{C,i}) = \frac{1}{2} \log_2 \left(1 + \frac{\left(\gamma^2\alpha^2 + (\gamma + \alpha)^2 + 1\right)P + \gamma^2 \omega_B + (1 + \gamma^2)\sigma^2}{\omega_C}\right) \leq C.$$
• Sum-rate maximization problem (P1)

$$\text{maximize } \frac{1}{2} \sum_{i \in \mathcal{N}} \log_2 \left( 1 + P \frac{1 + (\gamma + \alpha)^2 + \gamma^2 \alpha^2 + (\gamma + \alpha)(1 + \gamma \alpha) \lambda_{1,i} + \gamma \alpha \lambda_{2,i}}{\sigma^2 \gamma \lambda_{1,i} + \sigma^2 (\gamma^2 + 1) + \omega_B \gamma^2 + \omega_C} \right)$$

s.t. 
$$\frac{1}{2} \log_2 \left( 1 + \frac{P(1 + \alpha^2) + \sigma^2}{\omega_B} \right) \leq B,$$
$$\frac{1}{2} \log_2 \left( 1 + \frac{\left( \gamma^2 \alpha^2 + (\gamma + \alpha)^2 + 1 \right) P + \gamma^2 \omega_B + (1 + \gamma^2) \sigma^2}{\omega_C} \right) \leq C.$$
At optimal point, the capacity constraints should be tight.

Without loss of optimality, we can set

\[ \omega_B = \beta_B \left( P(1+\alpha^2) + \sigma^2 \right), \]
\[ \omega_C = \beta_C \left( (\gamma^2 \alpha^2 + (\gamma + \alpha)^2 + 1) P + \gamma^2 \omega_B + (1 + \gamma^2)\sigma^2 \right), \]

with \( \beta_B = 1/(2^{2B} - 1) \) and \( \beta_C = 1/(2^{2C} - 1) \).

Therefore, the optimal value for (P1) can be found via one-dimensional search over the coefficient \( \gamma \).
For reference, we consider the Cut-Set upper bound on $R_{\text{sum}}$ as

$$R_{\text{sum}} \leq \min \{ NC, R_{\text{full}} \},$$

where $R_{\text{full}}$ is the sum-rate achievable when full cooperation among RUs is possible, i.e.,

$$R_{\text{full}} = I \left( \{ X_i \}_{i \in \mathcal{N}} ; \{ Y_i \}_{i \in \mathcal{N}} \right)$$

$$= \frac{1}{2} \log_2 \det \left( I + P \left( \sigma^2 H_Z H_Z^T \right)^{-1} H_X H_X^T \right).$$
Numerical Example

- Per-UE rate versus RU-RU capacity $B$
  - $N = 3$, $\text{SNR} = 20 \text{ dB}$, $\alpha = 0.7$

  - With INP, the performance approaches upper bound as $B$ increases.
In this strategy, the quantized signals $\hat{Y}_{B,i}$ and $\hat{Y}_{C,i}$ are decompressed while leveraging (WZ-style) side information.

Decompression for RU-RU links
- Uplink received signal can be leveraged as side information.
- As long as inter-cell channel gain $\alpha > 0$

Decompression for RU-CU links
- Suppose successive decompression of $\hat{Y}_{C,1}, \hat{Y}_{C,2}, \ldots, \hat{Y}_{C,N}$.
- Then, previously decompressed signals can be leveraged as side information.
Decompression of $\hat{Y}_{B,i}$ at RU $[i+1]$

- Leveraging side information $Y_{[i+1]}$
- Constraint on $\omega_{B,i}$ [ElGamal-Kim, Ch. 10]

$$I(Y_i; \hat{Y}_{B,i} \mid Y_{[i+1]}) = \frac{1}{2} \log_2 \left( 1 + \frac{E[Y_i^2 \mid Y_{[i+1]}]}{\omega_{B,i}} \right) \leq B,$$

with $E[Y_i^2 \mid Y_{[i+1]}] = (1 + \alpha^2)P + \sigma^2 - \frac{\alpha^2 P^2}{(1 + \alpha^2)P + \sigma^2}$. 

Side information

![Diagram of communication network with RU connections and side information]
**Numerical Example**

- **Per-UE rate versus RU-RU capacity** $B$
  - $N = 3$, SNR = 20 dB, $\alpha = 0.7$

- With INP, the performance approaches upper bound as $B$ increases.
- Leveraging SI for RU-RU link provides a slight sum-rate gain.
Decompression of $\hat{Y}_{C,1}, \hat{Y}_{C,2}, \ldots, \hat{Y}_{C,N}$ at CU

- Consider a successive decompression with order $\hat{Y}_{C,1} \rightarrow \hat{Y}_{C,2} \rightarrow \ldots \rightarrow \hat{Y}_{C,N}$
- Condition on $\omega_{B,i}$

$$I(S_i; \hat{Y}_{C,i} \mid \hat{Y}_{C,i-1}) = \frac{1}{2} \log_2 \left( 1 + \frac{E[S_i^2 \mid \hat{Y}_{C,i-1}]}{\omega_C} \right) \leq C,$$

with $E[S_i^2 \mid \hat{Y}_{C,i-1}] = \left( \gamma_i^2 \alpha^2 + (\gamma_i + \alpha)^2 + 1 \right) P + \gamma_i^2 \omega_b + (1 + \gamma_i^2) \sigma^2$

$$- \frac{\left( \gamma_i \alpha \gamma \alpha (\gamma_{i-1} + \alpha) P + \gamma_i \sigma^2 \right)^2}{\left( \gamma_i^2 \alpha^2 + (\gamma_i + \alpha)^2 + 1 \right) P + \gamma_i^2 \omega_b + (1 + \gamma_i^2) \sigma^2}.$$
Numerical Example

- Per-UE rate versus RU-RU capacity $B$
  - $N = 3$, SNR = 20 dB, $\alpha = 0.7$

- With INP, the performance approaches upper bound as $B$ increases.
- Leveraging SI for RU-RU link provides a slight sum-rate gain.
- Leveraging SI for RU-CU link leads to a significant sum-rate gain especially for small $B$. 
Numerical Example

- Per-UE rate versus SNR $P/\sigma^2$
  - $N = 3$, $\alpha = 0.7$, $C = B \in \{1, 2\}$

When side information is leveraged, the impact of INP is not so significant.

There is still a large gap between the upper bound and the sum-rate performance.

Can the gap be reduced by joint decompression and decoding at CU (Noisy network coding)?

[Sanderovich et al:TIT][Lim et al:TIT]
[Park et al:SPL]

This is the optimal oblivious processing [Aguerri et al:arXiv].
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Joint decompression and decoding (JDD)

- Potentially larger rates can be achieved with JDD at CU [Sanderovich et al: TIT][Lim et al: TIT][Park et al: SPL].
  - Now often seen as an instance of noisy network coding [Lim et al: TIT].
  - Optimal oblivious processing [Aguerri et al: arXiv]
Joint Decompression and Decoding

- Joint decompression and decoding (JDD)
  - Achievable sum-rate under JDD for given $\omega_B$, $\omega_C$, $\gamma$ [Lim et al:TIT]

$$R_{\text{sum}} = \min_{\mathcal{S} \subseteq \mathcal{N}} \left\{ |\mathcal{S}| C - \sum_{i \in \mathcal{S}} I\left(S_i; \hat{Y}_{C,i}\right) + I\left(X; \{\hat{Y}_{C,i}\}_{\mathcal{N}\setminus\mathcal{S}}\right) \right\}$$

$$= \min_{\mathcal{S} \subseteq \mathcal{N}} \left\{ |\mathcal{S}| (C - \tilde{g}_C(\omega_B, \omega_C, \gamma)) + f_{C,\mathcal{S}}(\omega_B, \omega_C, \gamma) \right\},$$

where \( \tilde{g}_C(\omega_B, \omega_C, \gamma) = \frac{1}{2} \log_2 \left( 1 + \frac{\gamma^2 \omega_B + (1 + \gamma^2) \sigma^2}{\omega_C} \right), \)

\[ f_{C,\mathcal{S}}(\omega_B, \omega_C, \gamma) = \frac{1}{2} \log_2 \det \left( I + P \left( \sigma^2 H_{Z,\mathcal{S}} H_{Z,\mathcal{S}}^T + \omega_B H_{Q,\mathcal{S}} H_{Q,\mathcal{S}}^T + \omega_C I \right)^{-1} H_{X,\mathcal{S}} H_{X,\mathcal{S}}^T \right), \]

$H_{X,\mathcal{S}}$, $H_{Z,\mathcal{S}}$, $H_{Q,\mathcal{S}}$: Submatrices of $H_X$, $H_Z$, $H_Q$ with rows in $\mathcal{S}$ removed.
Numerical Example

- **Per-UE rate versus RU-RU capacity** $B$
  - $N = 3$, SNR = 20 dB, $\alpha = 0.7$

- With INP, the performance approaches upper bound as $B$ increases.
- Leveraging SI for RU-RU link provides a slight sum-rate gain.
- Leveraging SI for RU-CU link leads to a significant sum-rate gain especially for small $B$.
- JDD further improves the sum-rate performance.
Numerical Example

- Per-UE rate versus SNR $P/\sigma^2$
  - $N = 3$, $\alpha = 0.7$, $C = B \in \{1, 2\}$
  - JDD shows slightly improved performance, but the gap to upper bound is still large.
  - This calls for the development of
    - Improved scheme based on
      - Non-oblivious RU processing
    - Improved upper bound
      - Extending the idea as [Wu et al:arXiv]
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Concluding Remarks

- We have studied the role of inter-RU links for improving the sum-rate of C-RAN uplink.
  - Under the assumptions of
    - Oblivious processing at RUs
    - Wyner-type Gaussian channel

- Future work
  - Non-oblivious processing at RUs
    - Compute-and-Forward based techniques [Aguerri-Zaidi][Hong-Caire]
    - Edge processing
  - Improved outer bounds over the cut-set bound, extending ideas as: [Wu et al:arXiv]
  - General uplink C-RAN networks
  - Downlink of C-RAN (Oblivious and Non-oblivious schemes)
    - Possibly with edge processing or edge caching
Thank you!
References


References


References


Sum-Rates for Wyner-type C-RAN Uplink with Inter-Connected Oblivious Radio Units

This study addresses the achievable sum-rate for the uplink of a cloud radio access network (C-RAN) operating in a linear Wyner-type topology. In the system, a set of radio units (RUs) is connected to a control unit (CU) by means of digital finite-capacity fronthaul links, and the messages sent by the users equipment (UEs) served by the RUs are jointly decoded at the CU based on the compressed baseband signals received on the fronthaul links. The potential advantages of utilizing the inter-RU links to improve the sum-rate performance is examined. In the considered strategy, each RU performs in-network processing of the uplink received signal and of the compressed baseband signal received from the adjacent RU, with the CU performing channel decoding incorporating the in-network processing output signals. A closed-form expression of the achievable sum-rate is derived assuming point-to-point compression, and analytic expressions for other advanced options are also provided in the presence of compression schemes leveraging side information. Numerical examples provide insights into the advantages of inter-RU communications and into the performance gap to sum-rate upper bounds.

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