State-Dependent Parallel Gaussian Channels With a State-Cognitive Helper

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***Abstract*—The state-dependent parallel channel with differently scaled states and a common state-cognitive helper is studied, in which two transmitters wish to send two messages to their corresponding receivers respectively over two parallel Gaussian subchannels. The two Gaussian channels are corrupted by the same but differently scaled states. The state is not known to the transmitters nor to the receivers, but known to a helper noncausally, which assists the receivers to cancel the state. Differently from previous studies that characterized the capacity region only in the infinite state power regime and under independent state corruption at the two receivers, this paper investigates the case under arbitrary state power and with the same but differently scaled states. An inner bound on the capacity region is derived and is compared to an outer bound. Then the channel parameters are partitioned into various cases, and segments on the capacity region boundary are characterized for each case.**

***Index Terms*—Dirty paper coding, , Gelf'and-Pinsker scheme, noncausal channel state information, parallel channel.**

I. INTRODUCTION

With the development of cellular systems, to support more users and higher transmission rates, non-orthogonal multi-user access (NOMA) has been intensively investigated, where interference cancellation is the key issue for the non-orthogonal transmission. In this paper, we investigate a type of state-dependent channels with helper, in which the state is not known to either transmitters or receivers, but is noncausally known to a state-cognitive helper. This model captures interference cancelation in various practical scenarios. For example, users in a multi-cell systems may be interfered by a base station located in other cells. Such a base station, being as the source that causes the interference, clearly knows the information of the interference (modeled by state) and can serve as a helper to help to cancel the interference. Alternatively, that base station can also convey the interference information to other base stations via the back haul network so that other base station can serve as helpers to cancel the interference. As a comparison, this type of state-dependent models differ from the original state-dependent channels studied in e.g., [1] and [2], in that the state-cognitive helper is not informed of the transmitters' messages, and hence its state cancellation strategies are necessarily independent from message encoding at the transmitters.

The basic state-dependent Gaussian channel with a helper was introduced by [3], in which the capacity in the infinite power regime was characterized and was shown to be achievable by lattice coding. The capacity under arbitrary state power was established for some special cases in [4]. As more models, some state-dependent MACs also fall into the type of state-dependent models with state-cognitive helpers. The state-dependent asymmetric multiple access channel (MAC) was studied in [5], in which an inner bound was derived using Gelfand-Pinsker coding for the state-cognitive user, and using the regular MAC scheme for the uninformed user. In [6], the MAC with two states and with each state is known at one transmitter was studied, and the lattice coding was used to derive achievable regions. In [7], this channel was further studied with an additional common message shared between the informed and the uninformed user. New lower and upper bounds were derived. In a recent work [8] on the state-dependent MAC, a new outer bound was derived which is tighter than the previous bounds. In [4], the state-dependent MAC with an additional helper was studied, and the partial/full capacity region was characterized under various channel parameters. In [9], the state-dependent broadcast channel with a helper was studied, and an achievable rate region was derived using single-bin dirty paper coding at the helper and successive cancellation at the receivers. The capacity region was characterized under certain channel parameters. Moreover, some state-dependent relay channel models can also be viewed as an extension of the state-dependent channel with a helper, where the relay serves the role of the helper by knowing the state information. In [10], the state-dependent relay channel with state non-causally available at the relay is considered. An achievable rate was derived using a combination of decode-and-forward, Gelfand-Pinsker binning and codeword splitting. And in [11], additional noiseless cooperation links with finite capacity were assumed between the transmitter and the relay, and various coding techniques were explored.

The most relevant work to this paper is [12], in which the state-dependent parallel channel with a helper was studied, for the regime with infinite state power and with two receivers being corrupted by two independent states. A time-sharing scheme was proved to be capacity achieving under certain channel parameters. In contrast, in this paper, the two receivers of the parallel channel are corrupted by the same but differently scaled states, and the state can take arbitrary power. In this case, the time-sharing scheme is no longer optimal.

Thus, in this paper, we derive an inner bound on the

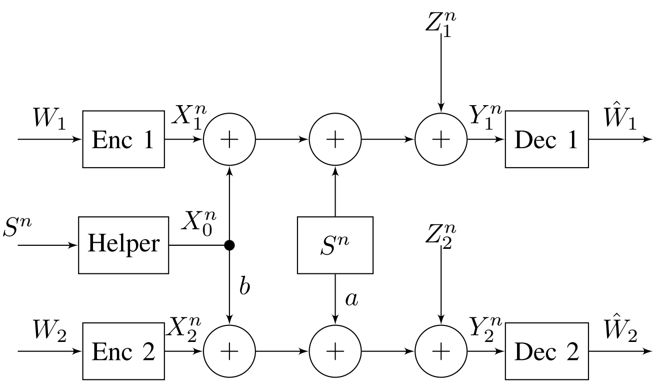


Fig. 1: The state-dependent parallel channel with a helper.

capacity region using an achievability scheme that integrates single-bin dirty paper coding and direct state subtraction. We then compare such an inner bound with an outer bound that consists of the capacity of point-to-point channel without state and an outer bound developed in [3] for the point-to-point state-dependent channel with a helper. The comparison yields the capacity region for certain ranges of channel parameters. More specifically, when the helper's power is above a certain threshold and the helper's signal is scaled the same as the state, we show that the state interference can be fully canceled for both channels, and thus the capacity region is the same as that of the corresponding channel without state.

II. Channel Model

In this paper, we study the state-dependent parallel network with a state-cognitive helper, in which two transmitters communicate with two corresponding receivers over a state-dependent parallel channel. The two receivers are corrupted by two differently scaled states, respectively. The state information is not know to either the transmitters or the receivers, but to a helper noncausally. Hence, the helper assists these receivers to cancel the state interference (see Figure 1).

More specifically, the encoder at transmitter i, $f_i : M_i \rightarrow \mathcal{X}_i^n$, maps a message $m_i \in \left\{1, \ldots, 2^{nR_i}\right\}$ to a codeword x_i^n, for $i = 1, 2$. The inputs x_1^n and x_2^n are sent respectively over the two subchannels of the parallel channel. The two receivers are corrupted by an independent and identically distributed (i.i.d.) state sequence $s^n \in \mathcal{S}^n$, which is known to a common helper noncausally. Hence, the encoder at the helper, $f_0 : \mathcal{S}^n \rightarrow \mathcal{X}_0^n$, maps the state sequence $s^n \in \mathcal{S}^n$ into a codeword $x_0^n \in \mathcal{X}_0^n$. The channel transition probability is given by $P_{Y_1|X_0X_1S} \cdot P_{Y_2|X_0X_2S}$. The decoder at receiver i, $g_i : \mathcal{Y}_i^n \rightarrow \mathcal{M}_i$, maps a received sequence y_i^n into a message $\hat{m}_i \in \mathcal{M}_i$, for $i = 1, 2$. We assume that the messages are uniformly distributed over the sets $\mathcal{M}_1$ and $\mathcal{M}_2$. We define the average probability of error for a length-n code as follows:

$$P_e = \frac{1}{|\mathcal{M}_1||\mathcal{M}_2|} \sum_{m_1=1}^{\mathcal{M}_1} \sum_{m_2=1}^{\mathcal{M}_2} \mathbb{P}\left\{\hat{m}_1 \neq m_1, \hat{m}_2 \neq m_2\right\}. \quad (1)$$

Definition 1. A rate pair (R_1, R_2) is said to be *achievable* if there exist a sequence of message sets $\mathcal{M}_1^{(n)}$ and $\mathcal{M}_2^{(n)}$ with $\left|\mathcal{M}_1^{(n)}\right| = 2^{nR_1}$ and $\left|\mathcal{M}_2^{(n)}\right| = 2^{nR_2}$, and encoder-decoder tuples $\left(f_0^{(n)}, f_1^{(n)}, f_2^{(n)}, g_1^{(n)}, g_2^{(n)}\right)$ such that the average probability of error $P_e^{(n)} \rightarrow 0$ as $n \rightarrow \infty$.

Definition 2. We define the *capacity region* of the channel as the closure of the set of all achievable rate pairs (R_1, R_2).

In this paper, we focus on the Gaussian channel, with the outputs at the two receivers for one channel use given by

$$Y_1 = X_0 + X_1 + S + Z_1 \quad (2a)$$

$$Y_2 = bX_0 + X_2 + aS + Z_2 \quad (2b)$$

where Z_1 and Z_2 are noise variables with Gaussian distributions $Z_1 \sim \mathcal{N}(0, 1)$ and $Z_2 \sim \mathcal{N}(0, 1)$, and S is the state variable with Gaussian distribution $S \sim \mathcal{N}(0, Q)$. Both the noise variables and the state variable are i.i.d. over channel uses. The channel inputs X_0, X_1, and X_2 are subject to the average power constraints $\frac{1}{n}\sum_{i=1}^n X_{0i}^2 \leq P_0$, $\frac{1}{n}\sum_{i=1}^n X_{1i}^2 \leq P_1$ and $\frac{1}{n}\sum_{i=1}^n X_{2i}^2 \leq P_2$. The constant a represents the channel gain of the state sequence in the second subchannel compared to the first subchannel. Similarly, the constant b is the gain of the helper signal in the second subchannel compared to that in the first subchannel. Thus our model presents a general scenario, where the helper's power and the state power can be arbitrary.

Our goal is to characterize the capacity region of the Gaussian channel under various channel parameters (a, b, P_0, P_1, P_2, Q).

III. Main Results

In this section, we first derive inner and outer bounds on the capacity region for the state-dependent parallel channel with a helper. Then by comparing the inner and outer bounds, we characterize the segments on the capacity region boundary under various different channel parameters. Some proofs are omitted due to space limitations.

A. Inner and Outer Bounds

We first derive an achievable region for the channel based on an achievable scheme that integrates direct state cancellation and single-bin dirty paper coding. More specifically, an auxiliary random variable (represented by U in Proposition 1) is generated to incorporate the state information so that each receiver jointly decodes the codeword in the bin and the message sent by the corresponding transmitter. Based on such an achievable scheme, we derive the following inner bound on the capacity region for the discrete memoryless channel.

Proposition 1. *For the discrete memoryless state-dependent parallel channel with a helper under the same but differently scaled states at the two receivers, an inner bound on the capacity region consists of rate pairs (R_1, R_2) satisfying:*

$$R_1 \leq \min\left\{I(U, X_1; Y_1) - I(U; S), I(X_1; Y_1|U)\right\} \quad (3a)$$

$$R_2 \leq \min\left\{I(U, X_2; Y_2) - I(U; S), I(X_2; Y_2|U)\right\} \quad (3b)$$

for some distribution

$$P_S P_{U|S} P_{X_0|US} P_{X_1} P_{X_2} P_{Y_1|SX_0X_1} P_{Y_2|SX_0X_2}.$$

We note that the first term in each rate of the above bound is based on the Gelfand-Pinsker binning scheme and joint decoding of the state and the message. The second term is based on successive cancellation of the channel state. More specifically, each receiver first decodes the helper's signal and then uses it to cancel the state, and then, the receiver decodes the desired signal.

Based on the above inner bound for the discrete memoryless case, we derive the following inner bound for the Gaussian channel.

Proposition 2. *An inner bound on the capacity region for the state-dependent parallel Gaussian channel with a helper and under the same but differently scaled state consists of rate pairs (R_1, R_2) satisfying:*

$$R_1 \leq \min\left\{f_{1,1}(\alpha,\beta,P_1), g_{1,1}(\alpha,\beta,P_1)\right\} \tag{4a}$$

$$R_2 \leq \min\left\{f_{a,b}(\alpha,\beta,P_2), g_{a,b}(\alpha,\beta,P_2)\right\} \tag{4b}$$

where, α and β are real constants satisfying $|\beta| \leq \sqrt{P_0/Q}$, and

$$f_{a,b}(\alpha,\beta,P) = \frac{1}{2}\log\frac{P_0'\left(b^2P_0' + (a+b\beta)^2Q + P + 1\right)}{P_0'Q(b\alpha - a - b\beta)^2 + P_0' + \alpha^2 Q}, \tag{5a}$$

$$g_{a,b}(\alpha,\beta,P) = \frac{1}{2}\log\left(1 + \frac{P\left(P_0' + \alpha^2 Q\right)}{P_0'Q(b\alpha - a - b\beta)^2 + P_0' + \alpha^2 Q}\right), \tag{5b}$$

where $P_0' = P_0 - \beta^2 Q$.

Proof. The proof follows from Proposition 1 by choosing the following joint Gaussian distribution for the random variables:

$$X_0 = X_0' + \beta S, \quad U = X_0' + \alpha S$$
$$X_0' \sim \mathcal{N}(0, P_0'), \quad X_1 \sim \mathcal{N}(0, P_1) \quad X_2 \sim \mathcal{N}(0, P_2)$$

where X_0', S, X_1, X_2 are independent. Furthermore, X_0 has the following power constraint:

$$P_0 \geq \mathbb{E}X_0^2 = P_0' + \beta^2 Q.$$

□

We note that the above choice of the helper's signal incorporates two parts with X_0' designed using single-bin dirty paper coding, and βS acting as state subtraction.

We next present an outer bound which applies the point-to-point channel capacity and the upper bound derived for the point-to-point channel with a helper in [3].

Lemma 1. *An outer bound on the capacity region of the states-dependent parallel Gaussian channel with a helper consists of rate pairs (R_1, R_2) satisfying:*

$$R_1 \leq \min\left\{\frac{1}{2}\log\left(1 + \frac{P_1}{P_0 + 2\rho_{0S}\sqrt{P_0Q} + Q + 1}\right) + \frac{1}{2}\log\left((1-\rho_{0S}^2)P_0 + 1\right), \frac{1}{2}\log\left(1 + P_1\right)\right\} \tag{6a}$$

$$R_2 \leq \min\left\{\frac{1}{2}\log\left(1 + \frac{P_2}{b^2P_0 + 2ab\rho_{0S}\sqrt{P_0Q} + a^2Q + 1}\right) + \frac{1}{2}\log\left((1-\rho_{0S}^2)b^2P_0 + 1\right), \frac{1}{2}\log\left(1 + P_2\right)\right\} \tag{6b}$$

for some ρ_{0S} that satisfies $-1 \leq \rho_{0S} \leq 1$.

B. Capacity Region Characterization

In this section, we optimize α and β in Proposition 2, and compare the rate bounds with the outer bounds in Lemma 1 to characterize the points or segments on the capacity region boundary.

We first define $\phi_{a,b}(\rho_{0S}, P)$ and $\theta_{a,b}(\rho_{0S}, P)$ as in (7) for notational convenience.

Since the inner bound in Proposition 2 is not convex, it is difficult to provide a close form for the jointly optimized bounds. Therefore, we first optimize the bounds for R_1 and R_2 respectively, and then provide conditions on channel parameters such that these bounds match the outer bound. Based on the conditions, we partition the channel parameters into the sets, in which different segments of the capacity region boundary can be obtained.

We first consider the rate bound for R_1 in (4a). By setting

$$\alpha_1 \triangleq \frac{(1+\beta_1)P_0'}{P_0'+1}, \quad \beta_1 \triangleq \rho_{0S}^*\sqrt{\frac{P_0}{Q}}$$

$f_{1,1}(\alpha,\beta,P_1)$ and $g_{1,1}(\alpha,\beta,P_1)$ take the following form

$$f_{1,1}(\alpha_1,\beta_1,P_1) = \phi_{1,1}(\rho_{0S}^*, P_1)$$
$$g_{1,1}(\alpha_1,\beta_1,P_1) = \theta_{1,1}(\rho_{0S}^*, P_1)$$

where $\rho_{0S}^* \in [-1, 1]$ maximizes $\phi_{1,1}(\rho_{0S}, P_1)$. In fact, α_1 maximizes $f_{1,1}(\alpha,\beta,P_1)$ for fixed β, and β_1 maximizes the function with $\alpha = \alpha_1$.

If $\phi_{1,1}(\rho_{0S}^*, P_1) \leq \theta_{1,1}(\rho_{0S}^*, P_1)$, $R_1 = \phi_{1,1}(\rho_{0S}^*, P_1)$ is achievable, and this matches the upper bound in (6a). Thus, one segment of the capacity region is specified by

$$R_1 = \phi_{1,1}(\rho_{0S}^*, P_1) \tag{8a}$$

$$R_2 = \min\{f_{a,b}(\alpha_1,\beta_1,P_2), g_{a,b}(\alpha_1,\beta_1,P_2)\} \tag{8b}$$

We further observe that the second term $g_{1.1}(\alpha,\beta,P_1)$ in (4a) is optimized by setting $\alpha = 1+\beta$, and hence

$$g_{1,1}(\alpha, \alpha-1, P_1) = \frac{1}{2}\log(1+P_1).$$

If $g_{1,1}(\alpha,\alpha-1,P_1) \leq f_{1,1}(\alpha,\alpha-1,P_1)$, i.e.,

$$P_0'^2 \geq \alpha^2 Q(P_1 + 1 - P_0'), \tag{9}$$

then the inner bound for R_1 becomes $R_1 = \frac{1}{2}\log(1+P_1)$, which is the capacity of the point-to-point channel without state and matches the outer bound in (6a). Thus one segment of the capacity is specified by

$$R_1 = \frac{1}{2}\log(1+P_1) \tag{10a}$$

$$R_2 = \min\{f_{a,b}(\alpha,\alpha-1,P_2), g_{a,b}(\alpha,\alpha-1,P_2)\}. \tag{10b}$$

$$\phi_{a,b}(\rho_{0S},P)=\frac{1}{2}\log\left(1+\frac{P}{b^2P_0+2ab\rho_{0S}\sqrt{P_0Q}+a^2Q+1}\right)+\frac{1}{2}\log\left((1-\rho_{0S}^2)b^2P_0+1\right) \tag{7a}$$

$$\theta_{a,b}(\rho_{0S},P)=\frac{1}{2}\log\left(1+\frac{P\left((1+b^2P_0(1-\rho_{0S}^2))^2+(1-\rho_{0S}^2)b^2P_0(a\sqrt{Q}+b\rho_{0S}\sqrt{P_0})^2\right)}{\left(a^2Q+2ab\rho_{0S}\sqrt{P_0Q}+b^2P_0+1\right)(b^2(1-\rho_{0S}^2)P_0+`1)}\right) \tag{7b}$$

We then consider the rate bound for R_2. Similarly, the following segments on the capacity boundary can be obtained. If $\phi_{a,b}(\rho_{0S}^{**},P_2)\le\theta_{a,b}(\rho_{0S}^{**},P_2)$, one segment of the capacity region boundary is specified by

$$R_1=\min\{f_{1,1}(\alpha_2,\beta_2,P_1),g_{1,1}(\alpha_2,\beta_2,P_1)\} \tag{11a}$$
$$R_2=\phi_{a,b}(\rho_{0S}^{**},P_2) \tag{11b}$$

where

$$\alpha_2\triangleq\frac{(a+b\beta_2)bP_0'}{b^2P_0'+1},\quad \beta_2\triangleq\rho_{0S}^{**}\sqrt{\frac{P_0}{Q}}$$

and $\rho_{0S}^{**}\in[-1,1]$ maximizes $\phi_{a,b}(\rho_{0S},P_2)$.

Furthermore, if $g_{a,b}(\alpha,\alpha-a/b,P_2)\le f_{a,b}(\alpha,\alpha-a/b,P_2)$, one segment of the capacity region boundary is specified by

$$R_1=\min\left\{f_{1,1}\left(\alpha,\alpha-\frac{a}{b},P_1\right),g_{1,1}\left(\alpha,\alpha-\frac{a}{b},P_1\right)\right\} \tag{12a}$$

$$R_2=\frac{1}{2}\log(1+P_2). \tag{12b}$$

Summarizing the above analysis, we obtain the following characterization of segments of the capacity region boundary.

Theorem 1. *The channel parameters (a,b,P_0,P_1,P_2,Q) can be partitioned into the sets $\mathcal{A}_1,\mathcal{B}_1,\mathcal{C}_1$, where*

$$\mathcal{A}_1=\{(a,b,P_0,P_1,P_2,Q):\phi_{1,1}(\rho_{0S}^*,P_1)\le\theta_{1,1}(\rho_{0S}^*,P_1)\}$$
$$\mathcal{C}_1=\{(a,b,P_0,P_1,P_2,Q):P_0'^2\ge\alpha^2Q(P_1+1-P_0')$$
$$\text{where } P_0'=P_0-(\alpha-1)^2Q,\text{ for some }\alpha\in\Omega_\alpha\}$$
$$\mathcal{B}_1=(\mathcal{A}_1\cup\mathcal{C}_1)^c.$$

If $(a,b,P_0,P_1,P_2,Q)\in\mathcal{A}_1$, then (8a) $-$ (8b) *captures one segment of the capacity region boundary, where the state cannot be fully cancelled. If $(a,b,P_0,P_1,P_2,Q)\in\mathcal{C}_1$, then* (10a) $-$ (10b) *captures one segment of the capacity region boundary where the state is fully cancelled. If $(a,b,P_0,P_1,P_2,Q)\in\mathcal{B}_1$, then the R_1 segment of the capacity region boundary is not characterized.*

The channel parameters (a,b,P_0,P_1,P_2,Q) can also be partitioned into the sets $\mathcal{A}_2,\mathcal{B}_2,\mathcal{C}_2$, where

$$\mathcal{A}_2=\{(a,b,P_0,P_1,P_2,Q):\phi_{a,b}(\rho_{0S}^{**},P_2)\le\theta_{a,b}(\rho_{0S}^{**},P_2)\}$$
$$\mathcal{C}_2=\{(a,b,P_0,P_1,P_2,Q):b^2P_0'^2\ge\alpha^2Q(P_2+1-b^2P_0')$$
$$\text{where } P_0'=P_0-\left(\alpha-\frac{a}{b}\right)^2Q,\text{ for some }\alpha\in\Omega_\alpha\}$$
$$\mathcal{B}_2=(\mathcal{A}_2\cup\mathcal{C}_2)^c.$$

If $(a,b,P_0,P_1,P_2,Q)\in\mathcal{A}_2$, then (11a) $-$ (11b) *captures one segment of the capacity region boundary, where the state cannot be fully cancelled. If $(a,b,P_0,P_1,P_2,Q)\in\mathcal{C}_2$, then* (12a) $-$ (12b) *captures one segment of the capacity boundary where the state is fully cancelled. If $(a,b,P_0,P_1,P_2,Q)\in\mathcal{B}_2$, then the R_2 segment of the capacity region boundary is not characterized.*

The above theorem describes two partitions of the channel parameters, respectively under which segments on the capacity region boundary corresponding to R_1 and R_2 can be characterized. Intersection of two sets, each from one partition, collectively characterizes the entire segments on the capacity region boundary.

Figure 2 lists all possible intersection of sets that the channel parameters can belong to. For each case in Figure 2, we use red solid line to represent the segments on the capacity region that are characterized in Theorem 1, and we also mark the value of the capacity that each segment corresponds to as characterized in Theorem 1.

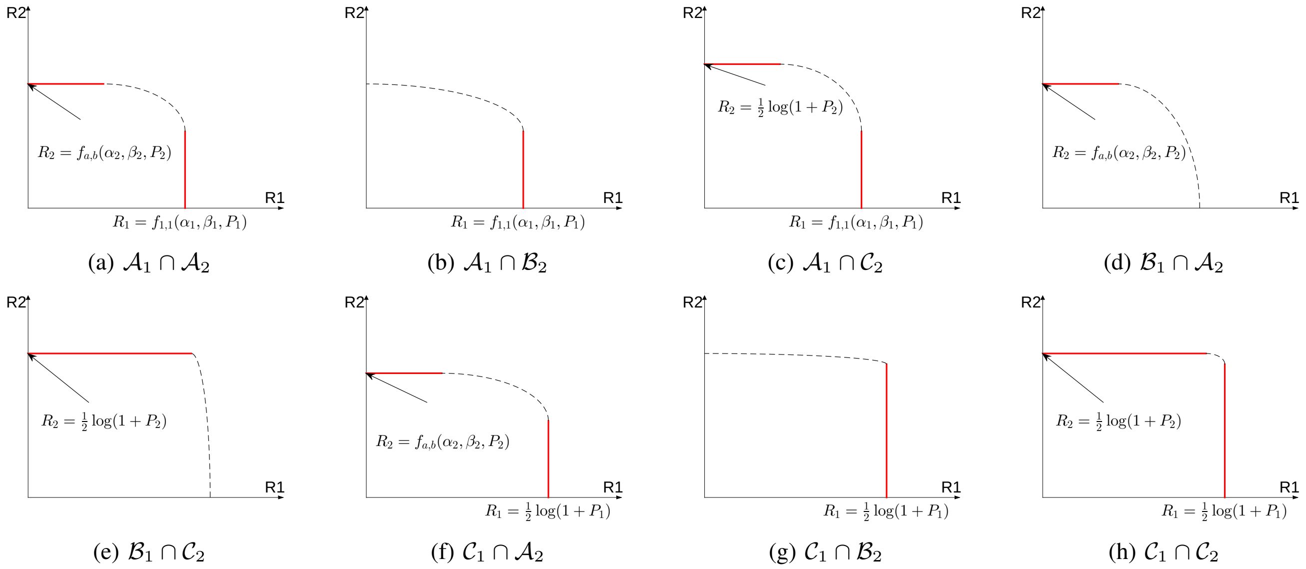
One interesting example in Theorem 1 is the case with $a=b$, in which R_1 and R_2 are optimized with the same set of coefficients α and β when $P_0'^2\ge\alpha^2Q(P_1+1-P_0')$ and $a^2P_0'^2\ge\alpha^2Q(P_2+1-a^2P_0')$. Thus, the point-to-point channel capacity is obtained for both R_1 and R_2, with state being fully cancelled. We state this result in the following theorem.

Theorem 2. *If $a=b$, $P_0'^2\ge\alpha^2Q(P_1+1-P_0')$ and $a^2P_0'^2\ge\alpha^2Q(P_2+1-a^2P_0')$ where $P_0'=P_0-(\alpha-1)^2Q$, for some $\alpha\in\Omega_\alpha$ then the capacity region of the state-dependent parallel Gaussian channel with a helper and under the same but differently scaled states contains (R_1,R_2) satisfying*

$$R_1\le\frac{1}{2}\log(1+P_1)$$
$$R_2\le\frac{1}{2}\log(1+P_2).$$

C. Numerical Example

We now examine our results via simulations. We set $P_1=P_2=5$, $Q=12$, $a=0.5$ and $b=0.5$. We plot the lower and upper bounds for the sum rate of R_1+R_2. We plot three lower bounds, corresponding to three cases of α, i.e., $\alpha_1,\alpha=1+\beta$ and α_2, respectively. In each case we maximize over ρ_{0S} to maximize the achievable sum rate. We further plot the upper bound based on Lemma 1, where we also maximize ρ_{0S}. It can be observed from Figure 3 that lower bound 1 with $\alpha=\alpha_1$ matches the upper bound well when the helper's power is small, whereas lower bound 2 with $\alpha=1+\beta$ matches the upper bound well when the helper's power is large. Both observations corroborate the characterization of the capacity in Theorem 1.



(a) $\mathcal{A}_1 \cap \mathcal{A}_2$ (b) $\mathcal{A}_1 \cap \mathcal{B}_2$ (c) $\mathcal{A}_1 \cap \mathcal{C}_2$ (d) $\mathcal{B}_1 \cap \mathcal{A}_2$

(e) $\mathcal{B}_1 \cap \mathcal{C}_2$ (f) $\mathcal{C}_1 \cap \mathcal{A}_2$ (g) $\mathcal{C}_1 \cap \mathcal{B}_2$ (h) $\mathcal{C}_1 \cap \mathcal{C}_2$

Fig. 2: Segments of the capacity region for all cases of channel parameters.

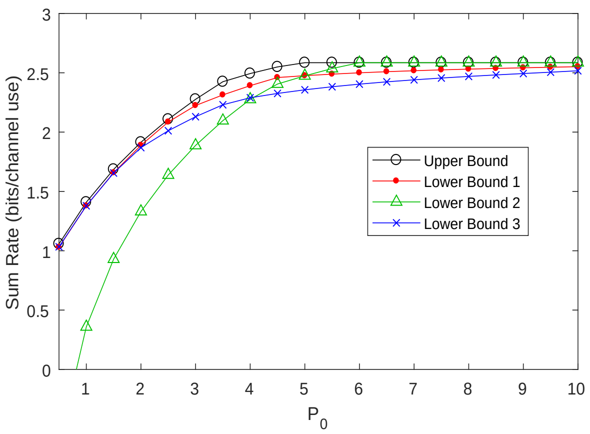


Fig. 3: Lower and upper bound on the sum capacity for the state-dependent parallel channel with a helper.

IV. Conclusion

In this paper, we have studied the parallel state-dependent Gaussian channel with a state-cognitive helper and with the same but differently scaled states. An inner bound was derived and was compared to an upper bound, and the segments of the capacity region boundary were characterized for various channel parameters. Furthermore, if the helper's signal and the state are equally scaled, the full rectangular capacity region of the two point-to-point channels without state can be achieved. As future work, we will analyze the case with channels being corrupted by independent states, and characterize the capacity region for various channel parameters.

Acknowledgment

The work of M. Dikshtein and S. Shamai was supported by the European Union's Horizon 2020 Research And Innovation Programme with grant agreement no. 694630, and by the Heron consortium via the Israel minister of economy and science. The work of Y. Liang was supported by NSF Grant CCF-1618127.

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