An Upper Bound on the Sum Capacity of the Downlink Multicell Processing with Finite Backhaul Capacity

Tianyu Yang, Nan Liu, Wei Kang, and Shlomo Shamai (Shitz)

***Abstract*—In this paper, we study upper bounds on the sum capacity of the downlink multicell processing model with finite backhaul capacity for the simple case of 2 base stations and 2 mobile users. It is modeled as a two-user multiple access diamond channel. It consists of a first hop from the central processor to the base stations via orthogonal links of finite capacity, and the second hop from the base stations to the mobile users via a Gaussian interference channel. The upper bound is derived using the converse tools of the multiple access diamond channel and that of the Gaussian MIMO broadcast channel. Through numerical results, it is shown that our upper bound improves upon the existing upper bound greatly in the medium backhaul capacity range, and as a result, the gap between the upper bounds and the sum rate of the time-sharing of the known achievable schemes is significantly reduced.**

I. Introduction

The multicell processing system, as reviewed in [1], has been used to increase the throughput and to cope with the inter-cell interference. The downlink multicell processing system, when first considered, consists of different base stations linked to the central processor via backhaul links of *unlimited* capacity, and therefore, the amount of cooperation among the different base stations is unbounded. This network can be modeled by a MIMO broadcast channel and the sum-rate characterization was found in [2]. Later on, due to the impracticality of unlimited capacity backhaul links, [3]–[7] studied the problem of finding the capacity region of the downlink multicell processing system when the capacities of the backhaul links are *finite*, and proposed various achievable schemes to efficiently utilize the finite capacity backhaul links. More specifically, in [3], a compressed dirty-paper coding scheme is proposed, where the base stations are treated as the antennas of the central processor and the dirty-paper coding codewords for each antenna are compressed and transmitted on the backhaul links. The scheme is improved in [4] by allowing the quantization noise of the base stations be correlated. The scheme of reverse compute-and-forward was proposed in [5] where linear precoding is performed at the central processor and the backhaul links are used to transmit linear combinations of the messages over a finite field. Such linear precoding transforms the channel seen at each mobile user into a point-to-point channel where integer-valued interference is eliminated by precoding and the remaining noninteger residual interference is treated as noise. By regarding the network model as a multi-user diamond channel, an achievability scheme is proposed in [6], [7] by combining Marton's achievability for the broadcast channel [8] and the achievability of sending correlated codewords over a multiple access diamond channel [9], [10].

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This work was partially supported by the National Natural Science Foundation of China under Grants 61571123, 61571122, and 61521061, the Research Fund of National Mobile Communications Research Laboratory, Southeast University (No. 2017A03) and Qing Lan Project. The work of S. Shamai was supported by the European Union's Horizon 2020 Research And Innovation Programme, grant agreement no. 694630.

The outer bound on the capacity region for this network is unknown except for the simple cut-set bound [11], which is the minimum of the capacity between the first hop from the central processor to the base stations and that of the second hop from the base stations to the mobile users. When the capacity of the backhaul links are relatively large, the performance of the scheme of compressed dirty-paper coding approaches that of the simple cut-set bound. On the other hand, when the capacity of the backhaul links are relatively small, the scheme of reverse compute-and-forward reaches the simple cut-set bound [6]. In the medium capacity region, there is still a relatively large gap between the simple cut-set upper bound and the performance of the time-sharing of the known achievable schemes. So it is unknown how well the proposed achievable schemes are and whether further efforts are needed in proposing better achievable schemes for the downlink multicell processing system.

In this paper, we derive a novel upper bound on the sum capacity of the downlink multicell processing network consisting of two base stations and two users. Similar to [6], we regard the network as a 2-user multiple access diamond channel. We first provide a cut-set upper bound using more cuts than the known simple cut-set bound of the minimum between the capacities of the first and the second hop. Next, single-letterization methods for the Gaussian multiple access diamond channel [12], [13] is applied to our problem. Finally, we obtain a novel upper bound on the sum capacity utilizing the converse tools of the Gaussian MIMO broadcast channel in [14]. The derived upper bound is expressed in terms of the sum capacity of the Gaussian MIMO broadcast channel given input covariance constraint, which has been found in [14]–[19], and thus, is easy to evaluate numerically.

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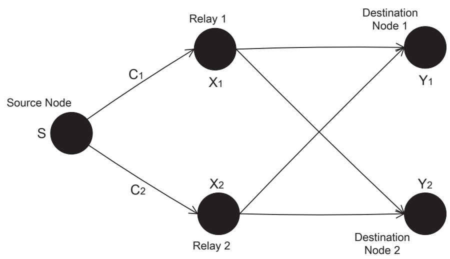

Fig. 1. The 2-user multiple access diamond channel.

Comparing numerically the proposed upper bound, the simple cut-set upper bound and the sum rate of various achievable schemes for the multicell processing system in terms of the sum-rate, we see that our upper bound improves upon the existing simple cut-set upper bound greatly in the medium backhaul capacity range, and as a result, the gap between the upper bounds and the sum rate of the time-sharing of the known achievable schemes is significantly reduced.

II. SYSTEM MODEL

In this paper, we consider the downlink multicell processing system with two base stations and two users. This network model can be seen as the 2-user multiple access diamond channel [6], see Fig. 1. The source node (central processor) can transmit to Relays (base stations) 1 and 2 via backhaul links of capacities C_1 and C_2, respectively. The channel between the two relay nodes and the two destination nodes (mobile users) is characterized by $p(y_1, y_2|x_1, x_2)$, with input alphabets $(\mathcal{X}_1, \mathcal{X}_2)$ and output alphabets $(\mathcal{Y}_1, \mathcal{Y}_2)$. Let W_1 and W_2 be two independent messages that the source node would like to transmit to Destinations 1 and 2, respectively. Assume that W_k is uniformly distributed on $\{1, 2, \cdots, M_k\}$, $k = 1, 2$.

An $(M_1, M_2, n, \epsilon_n)$ code consists of an encoding function at the source node:

$$f^n : \{1, 2, \cdots, M_1\} \times \{1, 2, \cdots, M_2\} \to \{1, 2, \cdots, 2^{nC_1}\} \times \{1, 2, \cdots, 2^{nC_2}\},$$

two encoding functions at the relay nodes:

$$f_k^n : \{1, 2, \cdots, 2^{nC_k}\} \to \mathcal{X}_k, \quad k = 1, 2,$$

and two decoding functions at the destination nodes:

$$g_k^n : \mathcal{Y}_k \to \{1, 2, \cdots, M_k\}, \quad k = 1, 2.$$

The average probability of error is defined as

$$\epsilon_n = \sum_{w_1=1}^{M_1} \sum_{w_2=1}^{M_2} \frac{1}{M_1 M_2} \Pr[g_1^n(Y_1^n) \neq w_1 \text{ or } g_1^n(Y_2^n) \neq w_2 | W_1 = w_1, W_2 = w_2].$$

Rate pair (R_1, R_2) is said to be achievable if there exists a sequence of $(2^{nR_1}, 2^{nR_2}, n, \epsilon_n)$ code such that $\epsilon_n \to 0$ as $n \to \infty$. The capacity region of the 2-user multiple access diamond channel is the closure of the set of all achievable rates pairs.

In this paper, we study the Gaussian case, where $\mathcal{X}_1 = \mathcal{X}_2 = \mathcal{Y}_1 = \mathcal{Y}_2 = \mathbb{R}$, and the channel between the two relays and each destination node is a Gaussian multiple access channel, i.e., the received signals at the destination nodes are

$$Y_1 = X_1 + aX_2 + U_1, \tag{1}$$

$$Y_2 = bX_1 + X_2 + U_2, \tag{2}$$

where X_1 and X_2 are the input signals from Relays 1 and 2, respectively, U_1, U_2 are two independent zero-mean unit-variance Gaussian random variables that are independent to (X_1, X_2), and $a, b \in \mathbb{R}$ are the channel gains from Relay 1 to Destination 2 and Relay 2 to Destination 1, respectively. Without loss of generality, we take $a \neq 0$ and $b \neq 0$. The case of $a = 0$ or $b = 0$ follows from continuity. The transmitted signals at the two relays must satisfy the average power constraints: for any x_k^n that Relay k sends into the channel, it must satisfy

$$\frac{1}{n} \sum_{i=1}^{n} x_{ki}^2 \leq P_k, \quad k = 1, 2.$$

III. AN UPPER BOUND ON THE SUM CAPACITY OF THE 2-USER GAUSSIAN MULTIPLE ACCESS DIAMOND CHANNEL

The following of this paper finds an upper bound on the sum capacity of the 2-user Gaussian multiple access diamond channel.

For $\bar{\rho} \in [-1, 1]$, define

$$C_{\text{MIMO}}^{\text{sum}}(\bar{\rho}) \triangleq \max_{(R_1, R_2) \in \mathcal{C}_{\text{MIMO}}(\bar{\rho})} R_1 + R_2, \tag{3}$$

where $\mathcal{C}_{\text{MIMO}}(\bar{\rho})$ denotes the capacity region of the broadcast channel described in (1) and (2) where $\mathbf{X} \triangleq \begin{bmatrix} X_1 & X_2 \end{bmatrix}^T$ is the transmitted signal of the 2 antennas of the transmitter, and Y_1 and Y_2 are the received signals of the single-antenna Receivers 1 and 2, respectively. The input of the transmitter must satisfy a covariance constraint, i.e.,

$$E\left[\mathbf{X}\mathbf{X}^T\right] \preceq \begin{bmatrix} P_1 & \bar{\rho}\sqrt{P_1 P_2} \\ \bar{\rho}\sqrt{P_1 P_2} & P_2 \end{bmatrix}.$$

The capacity region of the MIMO broadcast channel, i.e., $\mathcal{C}_{\text{MIMO}}(\bar{\rho})$, has been found in [14]–[16]. $C_{\text{MIMO}}^{\text{sum}}(\bar{\rho})$ defined in (3) is the sum capacity of the corresponding MIMO broadcast channel and it has been found in [17]–[19].

Before we introduce the main theorem, let us define the following functions of $\rho \in [-1, 1]$,

$$f_A(\rho) \triangleq C_1 + \frac{1}{2} \log\left(1 + \max\{a^2, 1\}(1 - \rho^2)P_2\right),$$

$$f_B(\rho) \triangleq C_2 + \frac{1}{2} \log\left(1 + \max\{b^2, 1\}(1 - \rho^2)P_1\right),$$

$$f_C(\rho) \triangleq C_1 + C_2 - \frac{1}{2} \log \frac{1}{1 - \rho^2},$$

and the following variables

$$\rho_x = \mathrm{sgn}(x)\left(\sqrt{1+\frac{1}{4x^2P_1P_2}}-\sqrt{\frac{1}{4x^2P_1P_2}}\right), \quad x=a,b,$$

where $\mathrm{sgn}(\cdot)$ is the sign function of $\cdot$, and the following sets

$$\mathcal{A}_x = \begin{cases} [0,\rho_x] & \text{if } x \geq 0 \\ [\rho_x, 0] & \text{if } x < 0 \end{cases}, \quad x=a,b.$$

The following is the main result of this paper.

Theorem 1 *The sum-rate R_1+R_2 is achievable for the 2-user Gaussian multiple access diamond channel only if it satisfies*

$$R_1+R_2 \leq \max\{\max_{\rho\in[-1,1]\bigcap \mathcal{A}_x^c} T_1(\rho), \quad \max_{\rho\in\mathcal{A}_x} T_2(\rho)\}, \tag{4}$$

for both $x=a$ and $x=b$, where $T_1(\rho)$ and $T_2(\rho)$ are defined as

$$T_1(\rho) = \min\left\{f_A(\rho), f_B(\rho), f_C(0), C_{MIMO}^{sum}(\rho)\right\}, \tag{5}$$

$$T_2(\rho) = \min\left\{T_1(\rho), \frac{1}{2}\left(f_C(\rho)+C_{MIMO}^{sum}(\rho)\right)\right\}, \tag{6}$$

Proof: Due to space limitations, the outline of the proof is provided in the appendix. For details of the proof, please refer to [20]. ∎

In Theorem 1, the upper bound of $T_1(\rho)$ is proved using the cut-set bound from the four cuts, i.e., Cuts A, B, C and D of Fig. 2, on the sum rate R_1+R_2. The more difficult part is to prove that when $\rho\in\mathcal{A}_x$, $x=a,b$, we have another upper bound of $\frac{1}{2}\left(f_C(\rho)+C_{\mathrm{MIMO}}^{\mathrm{sum}}(\rho)\right)$, which makes $T_2(\rho)$ strictly tighter than $T_1(\rho)$. The converse techniques we use to prove this include 1) the bounding of the correlation between the transmitted signals of the two relays via an auxiliary random variable [12], [13], which was inspired by Ozarow in solving the Gaussian multiple description problem [21]; 2) the single-letterization technique from [22, page 314, equation (3.34)]; 3) the entropy power inequality (EPI) [23, Lemma I]; and 4) the derivation of the capacity region of the Gaussian MIMO broadcast channel with private messages in [14, Section III.A].

The existing simple cut-set upper bound on the sum capacity is

$$R_1+R_2 \leq \min\{f_C(0), \max_{\rho\in[-1,1]} C_{\mathrm{MIMO}}^{\mathrm{sum}}(\rho)\}, \tag{7}$$

which is the minimum of the capacity of Cuts C and D of Fig. 2. Comparing this with the result of Theorem 1, we see that Theorem 1 implies that the new cut-set upper bound of

$$R_1+R_2 \leq \max_{\rho\in[-1,1]} T_1(\rho) \tag{8}$$

is true since we have $T_1(\rho)\geq T_2(\rho)$ for $\forall\rho\in[-1,1]$. The upper bound of (8) is tighter than the existing simple cut-set bound of (7), as it further considers the capacities of Cuts A and B. Furthermore, the result of Theorem 1, i.e., (4), is strictly tighter than the cut-set bound in (8) because when $\rho\in\mathcal{A}_x$, $x=a,b$, an upper bound of $T_2(\rho)$ exists, which is smaller than $T_1(\rho)$. Thus, Theorem 1 provides a novel upper bound that is tighter than the existing simple cut-set bound of (7).

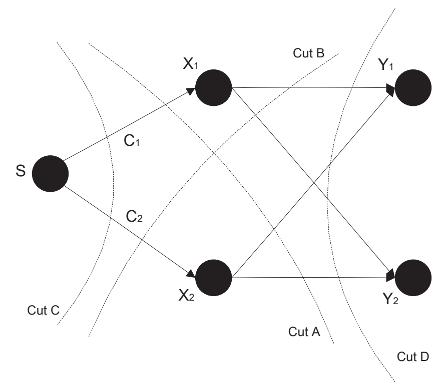

Fig. 2. Cut-set bounds for the channel

IV. Numerical Results

To illustrate the tightness of the derived upper bound in Theorem 1, we plot and compare the existing simple cut-set upper bound on the sum capcity in (7), the new cut-set upper bound of (8), the new upper bound of Theorem 1, and the achievable sum rates of existing schemes for the 2-user Gaussian multiple access diamond channel.

The results are shown in Fig. 3 for the symmetric case of $a=b=0.9$, $P_1=P_2=10$ and $C_1=C_2=C$. We only plot the region of $C\in[1,3]$, since this is the interesting case where the existing simple cut-set upper bound and the existing lower bounds on the sum capacity do not meet. As can be seen, in the region of $C\in[1.2,2.55]$, the new cut-set bound of (8) improves upon the existing simple cut-set bound of (7), which means that in this region, it is beneficial to consider the cross-cuts in the cut-set bound, i.e., Cuts A and B. In the region of $C\in[1.05,2]$, the upper bound of Theorem 1 improves upon the new cut-set bound of (8), which means that in this region, the derived upper bound of $\frac{1}{2}\left(f_C(\rho)+C_{\mathrm{MIMO}}^{\mathrm{sum}}(\rho)\right)$ for $\rho\in\mathcal{A}_x$, $x=a,b$ is useful.. Overall, in the region of $C\in[1.05,2.55]$, our new upper bound improves upon the existing simple cut-set upper bound strictly. Furthermore, in the region of $C\in[1.05,2]$, the improvement is rather significant.

The sum rate achieved by the achievable schemes of sending correlated codewords by the relays [6] , the compressed dirty-paper coding allowing correlated quantization noise [4] and the reverse compute-and-forward scheme [5] are denoted by the solid, circled, and dashed lines, respectively. Furthermore, the sum rate of the time-sharing of all the existing achievable schemes, which is the largest known lower bound for the sum capacity, is denoted by the dot-dashed line. In the gap between the derived upper bound in Theorem 1, i.e., the diamond line, and the largest known lower bound for the sum capacity, i.e., the dot-dashed line, lies the sum capacity of the 2-user Gaussian multiple access diamond channel for this symmetric case, and as we can see, the gap is not large, which means that the existing achievable schemes perform reasonably well for this scenario.

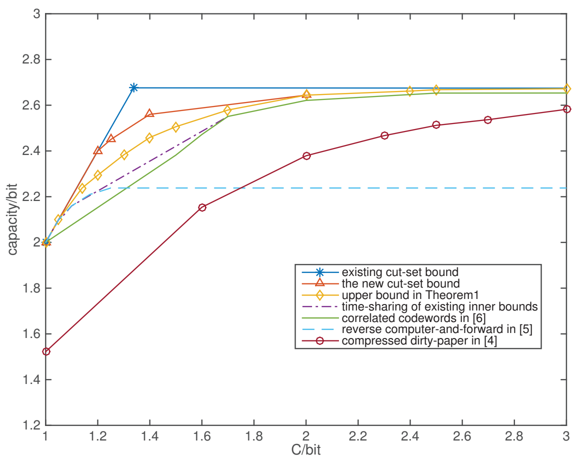

Fig. 3. Upper and lower bounds on the sum capacity for the case of $a = b = 0.9$, $P_1 = P_2 = 10$ and $C_1 = C_2 = C$.

V. Conclusion

In this paper, we derive a novel upper bound on the sum capacity of the 2-user Gaussian multiple access diamond channel. This is done by utilizing the converse tools of the multiple access diamond channel and that of the Gaussian MIMO broadcast channel. Through numerical results, we show that the derived upper bound improves upon the existing simple cut-set upper bound significantly, and as a result, the gap between the lower and upper bounds on the sum capacity is greatly reduced when the capacities of the backhaul links are in the medium range.

Appendix

Proof outline of Theorem 1: For any sequence of $(2^{nR_1}, 2^{nR_2}, n, \epsilon_n)$ code, let X_k^n denote the input of Relay k into the n uses of the channel $p(y_1, y_2|x_1, x_2)$, and Y_k^n denote the corresponding output received at Receiver k, $k = 1, 2$. Due to the power constraint, we have

$$\frac{1}{n}\sum_{i=1}^{n} E[X_{ki}^2] \leq P_k, \quad k = 1, 2. \tag{9}$$

Define of a random variable Q that is independent of everything else and uniformly distributed on $\{1, 2, \cdots, n\}$, further define

$$X_1 \triangleq X_{1Q}, X_2 \triangleq X_{2Q}, Y_1 \triangleq Y_{1Q}, Y_2 \triangleq Y_{2Q}. \tag{10}$$

Define the correlation coefficient between X_1 and X_2 as $\rho \triangleq \frac{E[X_1X_2]}{\sqrt{E[X_1^2]E[X_2^2]}}$. Note that $\rho \in [-1, 1]$. Further define $\bar{P}_k \triangleq E[X_k^2]$, $\quad k = 1, 2$. From (9) and (10), we have

$$\bar{P}_k \leq P_k, \quad k = 1, 2. \tag{11}$$

Define $\rho^* \triangleq \frac{\sqrt{\bar{P}_1\bar{P}_2}}{\sqrt{P_1P_2}}\rho$. Based on (11), we have $|\rho^*| \leq |\rho|$. Hence, $\rho^* \in [-1, 1]$. Define $\mathbf{X} \triangleq \begin{bmatrix} X_1 & X_2 \end{bmatrix}^T$, and further define $\mathbf{K}$ as

$$\mathbf{K} \triangleq \begin{bmatrix} P_1 & \rho^*\sqrt{P_1P_2} \\ \rho^*\sqrt{P_1P_2} & P_2 \end{bmatrix}.$$

We can see that

$$E[\mathbf{X}\mathbf{X}^T] \preceq \mathbf{K}. \tag{12}$$

Based on the four cuts demonstrated in Fig. 2, we have the following cut-set upper bounds on the sum capacity :

$$R_1 + R_2 \leq \min\left[f_A(\rho^*), f_B(\rho^*), f_C(0), C_{\text{MIMO}}^{\text{sum}}(\rho^*)\right]. \tag{13}$$

where in obtaining (13), we have used similar derivations as that of the degraded broadcast channel [23]. Note that (13) is valid for all values of $\rho^* \in [-1, 1]$.

We now proceed to derive another upper bound on $R_1 + R_2$ which is valid when $\rho^* \in \mathcal{A}_b$. Using Fano's inequality and omitting the ϵ_n term, we have

$$\begin{aligned}
&2n(R_1 + R_2) \\
&\leq n(C_1 + C_2) - I(X_1^n; X_2^n) + I(X_1^n, X_2^n; Y_2^n) \\
&\quad + I(X_1^n, X_2^n; Y_1^n|W_2) - I(X_1^n, X_2^n; Y_2^n|W_2) \\
&\leq n(C_1 + C_2) + I(X_1^n, X_2^n; Y_2^n) - I(X_1^n, X_2^n; Z^n) \\
&\quad + I(X_1^n; Z^n|X_2^n) + I(X_2^n; Z^n|X_1^n) + I(X_1^n, X_2^n; Y_1^n|W_2) \\
&\quad - I(X_1^n, X_2^n; Y_2^n|W_2),
\end{aligned} \tag{14}$$

where (14) follows by introducing a sequence of auxiliary random variables Z^n, similar to [12], [13], and utilizing the fact that

$$\begin{aligned}
&I(X_1^n; X_2^n) = I(X_1^n; Z^n) - I(X_1^n; Z^n|X_2^n) + I(X_1^n; X_2^n|Z^n) \\
&\geq I(X_1^n; Z^n) - I(X_1^n; Z^n|X_2^n) \\
&= I(X_1^n, X_2^n; Z^n) - I(X_2^n; Z^n|X_1^n) - I(X_1^n; Z^n|X_2^n).
\end{aligned}$$

The above derivation is true for any Z^n.

Next, we perform the single-letterization of (14). To do this, we restrict ourselves to considering Z^n that is the output of the following memoryless Gaussian channel with Y_2^n being the input:

$$Z = Y_2 + U_3, \tag{15}$$

where U_3 is a Gaussian random variable with zero mean and variance N_3. Further define $Z \triangleq Z_Q$. We single-letterize (14) by single-letterizing each of the following three terms:

$$\begin{aligned}
&I(X_1^n, X_2^n; Y_2^n) - I(X_1^n, X_2^n; Z^n) \\
&\qquad \leq nI(X_1, X_2; Y_2|Z), \qquad (16) \\
&I(X_1^n; Z^n|X_2^n) + I(X_2^n; Z^n|X_1^n) \\
&\qquad \leq n(I(X_1; Z|X_2) + I(X_2; Z|X_1)), \qquad (17) \\
&I(X_1^n, X_2^n; Y_1^n|W_2) - I(X_1^n, X_2^n; Y_2^n|W_2) \\
&\qquad = n\left[I(X_1, X_2; Y_1|U) - I(X_1, X_2; Y_2|U)\right], \qquad (18)
\end{aligned}$$

where in obtaining (18), we have used [22, page 314, equation (3.34)] and defined $V_i \triangleq \left(W_2, Y_1^{i-1}, Y_{2(i+1)}^n\right)$, $V \triangleq V_Q$ and $U \triangleq (V, Q)$.

From (14), (16), (17), and (18), we obtain the following single-letterization:

$$2(R_1+R_2) \leq C_1+C_2+I(X_1,X_2;Y_2|Z)+I(X_1;Z|X_2) \\ +I(X_2;Z|X_1)+I(X_1,X_2;Y_1|U)-I(X_1,X_2;Y_2|U), \tag{19}$$

where the mutual informations are evaluated using the joint distribution of the defined random variables $(X_1, X_2, Y_1, Y_2, Z, U)$ which satisfies

$$p(x_1,x_2,y_1,y_2,z,u) \\ = p(x_1,x_2)p(u|x_1,x_2)p(y_1,y_2|x_1,x_2)p(z|y_2). \tag{20}$$

Next, we further derive an upper bound on (19) by using the fact that $p(y_1, y_2|x_1, x_2)$ in (20), which refers to the channel in (1) and (2), and $p(z|y_2)$ in (20), which refers to the channel in (15), are Gaussian channels. To derive an upper bound on (19), we provide an upper bound for the following terms.

$$I(X_1,X_2;Y_2|Z) \leq \frac{1}{2}\log\left(b^2P_1+P_2+2b\rho^*\sqrt{P_1P_2}+1\right) \\ -\frac{1}{2}\log\left(\frac{b^2P_1+P_2+2b\rho^*\sqrt{P_1P_2}+1+N_3}{1+N_3}\right), \tag{21}$$

$$I(X_1;Z|X_2) \leq \frac{1}{2}\log\frac{(1-\rho^{*2})b^2P_1+1+N_3}{1+N_3}, \tag{22}$$

$$I(X_2;Z|X_1) \leq \frac{1}{2}\log\frac{(1-\rho^{*2})P_2+1+N_3}{1+N_3}, \tag{23}$$

where (21) follows from the EPI [23, Lemma I]. Finally, for the term $I(X_1, X_2; Y_1|U) - I(X_1, X_2; Y_2|U)$, we have

$$I(X_1,X_2;Y_1|U)-I(X_1,X_2;Y_2|U) \\ \leq \sup_{p(u,x_1,x_2):E[\mathbf{X}\mathbf{X}^T]\leq\mathbf{K}} \left(I(X_1,X_2;Y_1|U)-I(X_1,X_2;Y_2|U)\right) \tag{24}$$

$$= C_{\text{MIMO}}^{\text{sum}}(\rho^*) - \max_{p(x_1,x_2):E[\mathbf{X}\mathbf{X}^T]\preceq\mathbf{K}} I(\mathbf{X};Y_2) \tag{25}$$

$$= C_{\text{MIMO}}^{\text{sum}}(\rho^*) - \frac{1}{2}\log(b^2P_1+P_2+1+2b\rho^*\sqrt{P_1P_2}), \tag{26}$$

where (24) follows because U, X_1 and X_2 defined satisfy the constraint of the optimization in (24) due to (12), and according to [14, Section III.A], with continuity, we have (25). From (19), (21), (22), (23) and (26), we have

$$2(R_1+R_2) = (C_1+C_2)+C_{\text{MIMO}}^{\text{sum}}(\rho^*) \\ +\frac{1}{2}\log\frac{((1-\rho^{*2})b^2P_1+1+N_3)((1-\rho^{*2})P_2+1+N_3)}{(1+N_3)(b^2P_1+P_2+2b\rho^*\sqrt{P_1P_2}+1+N_3)}. \tag{27}$$

The above is true for any $N_3 \geq 0$. When $\rho^* \in \mathcal{A}_b$, take N_3 as

$$N_3 = b\sqrt{P_1P_2}\left(\frac{1}{\rho^*}-\rho^*\right)-1. \tag{28}$$

which is non-negative. Plugging (28) into (27), we obtain

$$2(R_1+R_2) \leq f_C(\rho^*)+C_{\text{MIMO}}^{\text{sum}}(\rho^*). \tag{29}$$

Due to symmetry, we may swap the indices 1 and 2 and re-derive the formulas from (14) to (29), and obtain that when $\rho^* \in \mathcal{A}_a$ we again have (29). Thus, from (13) and (29), we have proved Theorem 1.

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