Semantically-Secured Message-Key Trade-off over Wiretap Channels with Random Parameters

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Joint work with:
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Outline

1. The Wiretap Channel (WTC) - Different metrics for security
2. Soft Covering
3. The Gelfand-Pinsker (GP) Channel - Analysis using Likelihood Enc.
4. The GP-WTC
5. Secret Key-Message Trade-Off over the GP-WTC

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★ The results are **existential** and **asymptotic**

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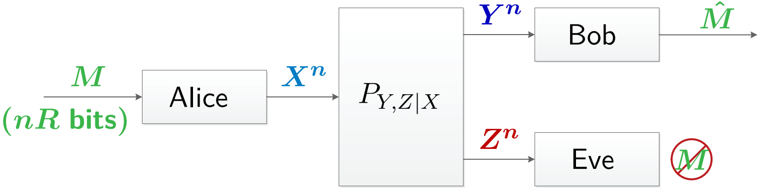
Degraded [Wyner 1975], General [Csiszár-Körner 1978]



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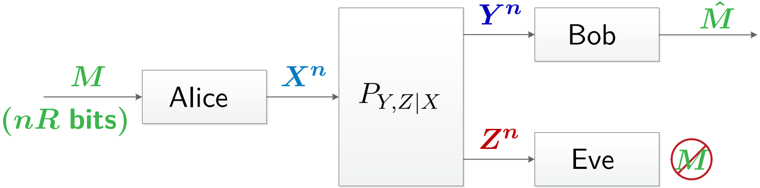

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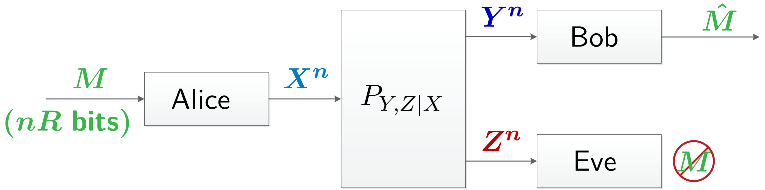

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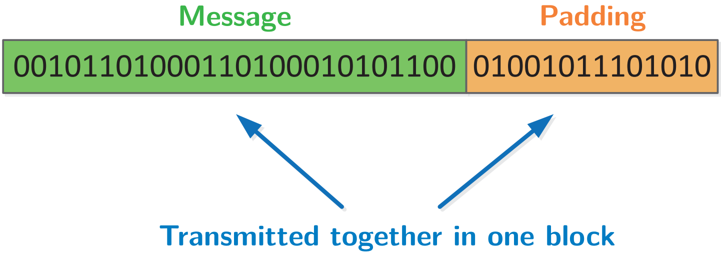
Theorem (Csiszár-Körner 1978)

$$\mathsf{C}_{\mathsf{WTC}} = \max_{P_{U,X}} \Big[I(U;Y) - I(U;Z) \Big]$$

Joint distribution: $P_{U,X} P_{Y,Z|X} \quad \Big\{ U -\!\!\circ\!\!- X -\!\!\circ\!\!- (Y,Z) \Big\}$

The Wiretap Channel - Encoding

- **Random codebook:** (Message, Padding) $\rightarrow$ $U^n \sim P_U^n$.
- **Codebook binning:** Pad nR **message bits** with $n\tilde{R}$ **random bits**.



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- **Security:** $\tilde{R} > I(U;Z)$.

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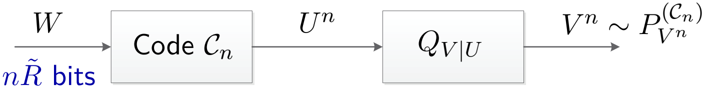

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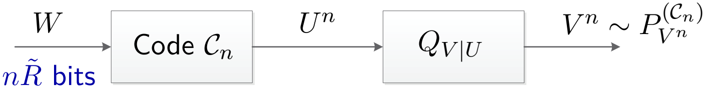

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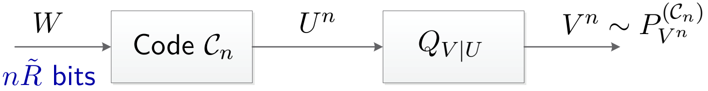

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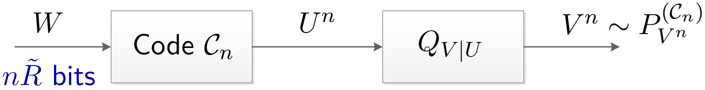

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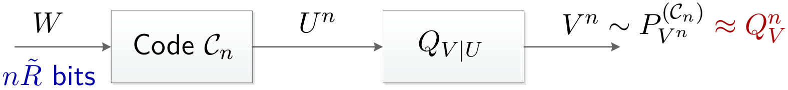

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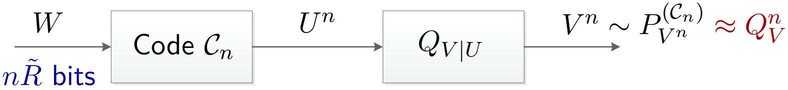

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- **Goal:** Choose $\tilde{R}$ (codebook size) s.t. $P_{V^n}^{(\mathcal{C}_n)} \approx Q_V^n$.

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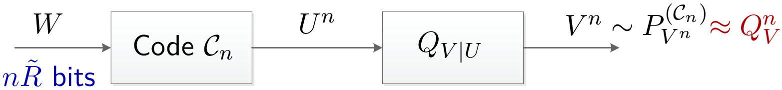

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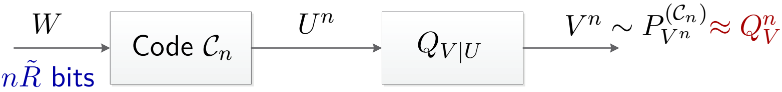

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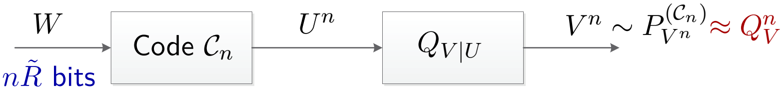

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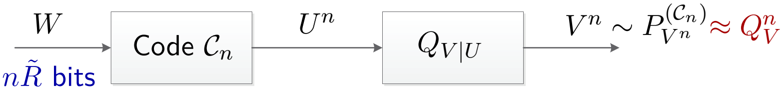

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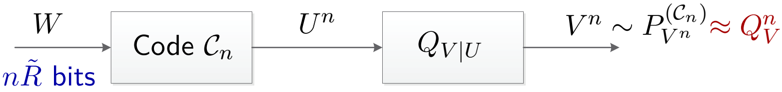

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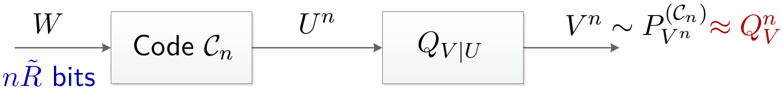

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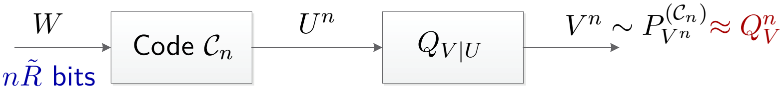

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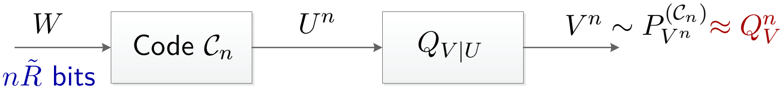

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Semantic Security

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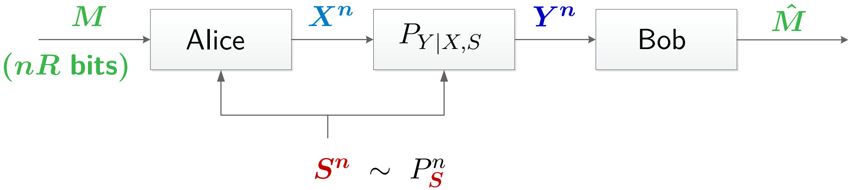
$$\Longrightarrow \quad \boxed{R < I(U;Y) - I(U;Z)}$$

The Gelfand-Pinsker Channel

[Gelfand-Pinsker 1980]



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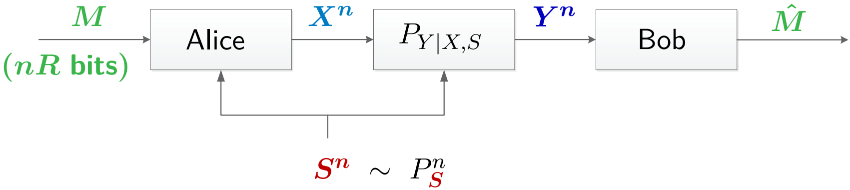
[Gelfand-Pinsker 1980]



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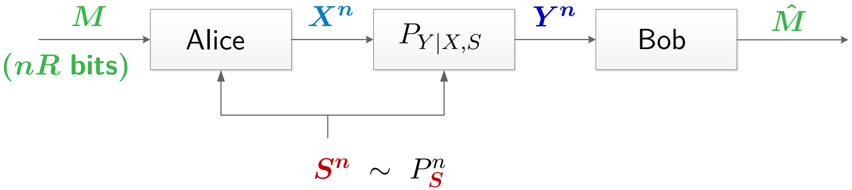

Capacity: Reliable communication.

Theorem (Gelfand-Pinsker 1980)

$$\mathsf{C}_{\mathsf{GP}} = \max_{P_{U,X|S}} \Big[I(U;Y) - I(U;S) \Big]$$

Joint distribution: $P_{U,X|S} P_{Y|X,S}$ $\Big\{ U - (X,S) - Y \Big\}$

The Gelfand-Pinsker Channel - Encoding

- **Random codebook:** (Message, Padding) $\rightarrow$ $U^n \sim P_U^n$.
- **Codebook binning:**
  Pad nR **message bits** with $n\tilde{R}$ **skillfully chosen bits**.

Message	Padding
00101101000110100010101100	01001011101010

Transmitted together in one block

The **message** chooses the bin, and the **padding** a specific codeword.

The Gelfand-Pinsker Channel - Analysis

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The Gelfand-Pinsker Channel - Analysis

Alternative Analysis - Likelihood Encoder + SCL:

- For simplicity assume: $U = X$

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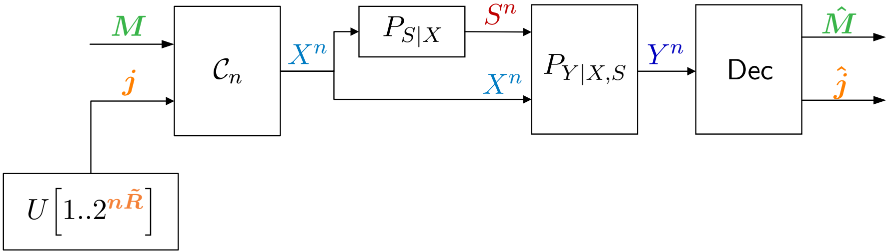

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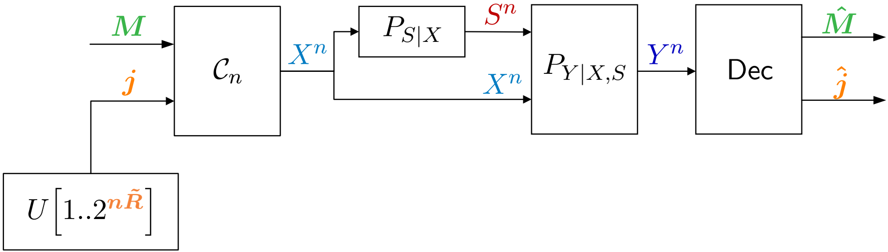

- Difference from original distribution: $\tilde{P}^{(\mathcal{C}_n)}_{\boldsymbol{J},\boldsymbol{S^n}|M}$.

The Gelfand-Pinsker Channel - Analysis
Alternative Analysis

- Define the encoder by the marginal $\tilde{P}^{(\mathcal{C}_n)}_{\boldsymbol{J}|M,\boldsymbol{S^n}}$
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$$P^{(\mathcal{C}_n)}_{\boldsymbol{J}|M,\boldsymbol{S^n}}(\boldsymbol{j}|\boldsymbol{m},\boldsymbol{s^n}) = \frac{P^n_{\boldsymbol{S}|\boldsymbol{X}}(\boldsymbol{s^n}|\boldsymbol{x^n}(\boldsymbol{m},\boldsymbol{j}))}{\sum_{j'} P^n_{\boldsymbol{S}|\boldsymbol{X}}(\boldsymbol{s^n}|\boldsymbol{x^n}(\boldsymbol{m},j'))}$$

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⋆ In standard analysis **Joint Typicality Encoder** is used instead.

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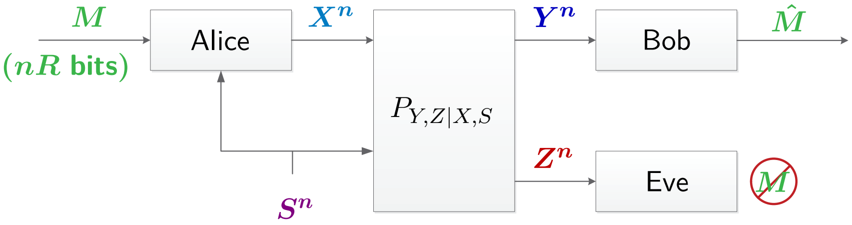
★ Ziv Goldfeld (MIT, USA); Haim H Permuter (Ben-Gurion University, Israel), "A Useful Analogy Between Wiretap and Gelfand-Pinsker Channels," ISIT 2018, Vail, Colorado.

The Gelfand-Pinsker Wiretap Channel



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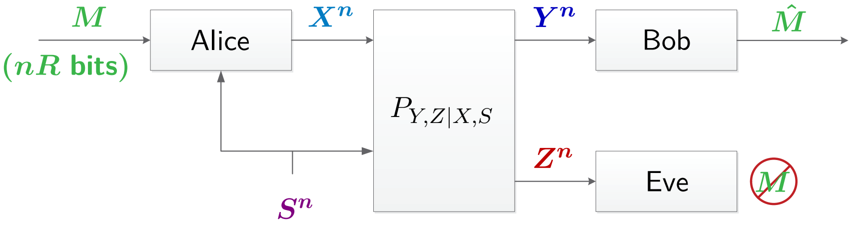
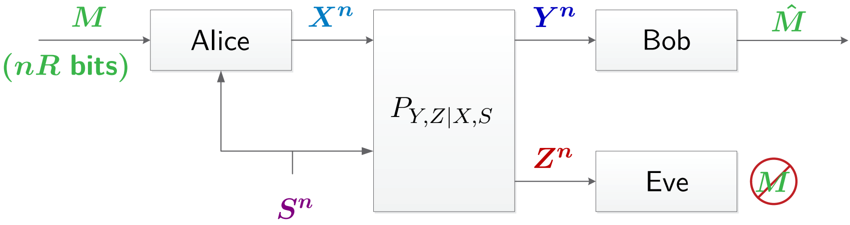


Secrecy-Capacity:

- Reliable Communication.
- Z^n contains no information about M.

The Gelfand-Pinsker Wiretap Channel



The State Information Plays a Double Role:

- Enhancing the total reliable communication rate
- Enhancing Bob's advantage over Eve

The Gelfand-Pinsker Wiretap Channel
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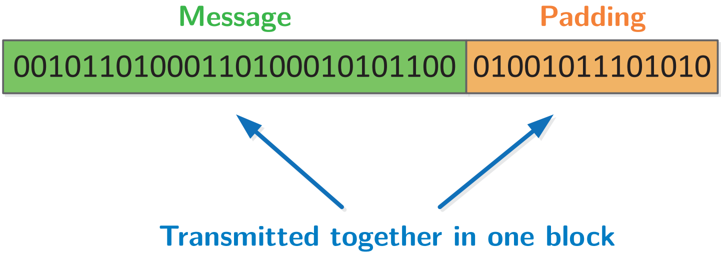

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$$\overbrace{\boxed{00101101000110100010101100}}^{nR}\overbrace{\underbrace{\boxed{01001011}}_{nI(U;Z)}\underbrace{\boxed{101010}}_{n\tilde{R}_2}}^{nI(U;S)}$$

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  - Secure
  - Uniformly distributed
  - Not (necessarily) controllable

The Gelfand-Pinsker Wiretap Channel

Theorem (Prabhakaran *et al.* 2012 – Special Case)

A secret message-key rate couple $(\boldsymbol{R_m}, \boldsymbol{R_k})$ *such that:*

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Proposition - Achievable Secret Key Rate ($R_m = 0$) (Khisti 2010)

$$\mathsf{C}^{\mathsf{SK}}_{\mathsf{GP-WTC}} \geq \max_{\substack{P_{U,X|S}: \\ I(U;Y)-I(U;S)\geq 0}} \Big[\boldsymbol{I(U;Y)} - \boldsymbol{I(U;Z)} \Big]$$

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The GP WTC - Two Layer Scheme

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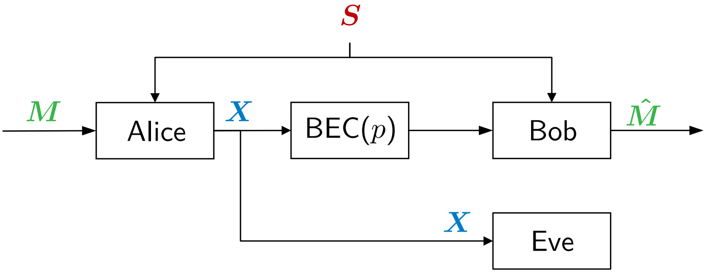

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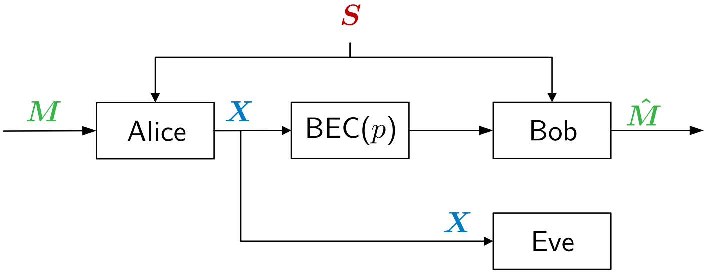

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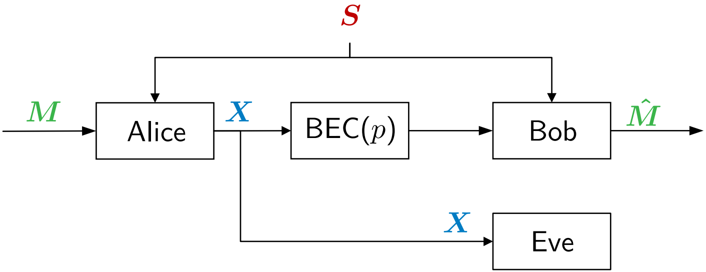

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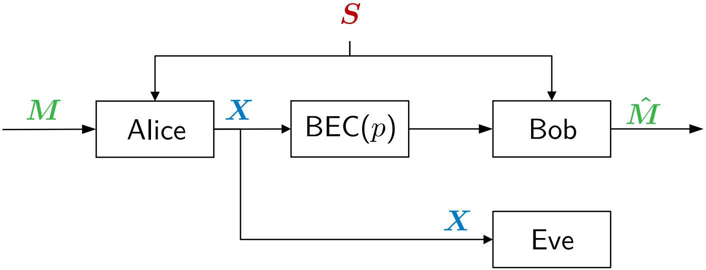

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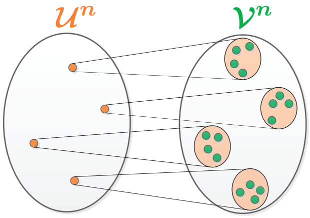
2. Enhance reliability without harming security by adding an inner coding layer.

The GP WTC - Two Layer Scheme

Superposition Code:



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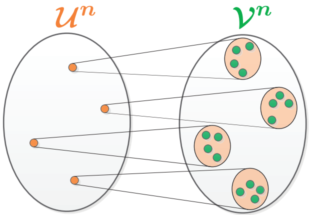

- U^n is decodable by **Eve** with redundancy $\Longrightarrow$ waste channel resources.
- All secrecy comes from V^n.

The GP WTC - Two Layer Scheme

Theorem (Prabhakaran *et al.* 2012)

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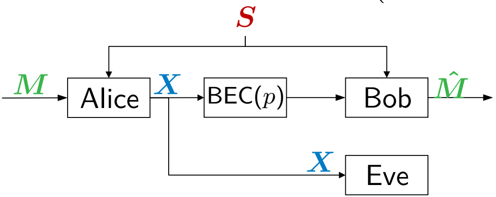

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- **Weak** secrecy only!

The GP WTC - Two Layer Scheme

Theorem (Goldfeld *et al.* 2016)

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Our results

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- **Insufficient resolution in the inner layer reduces secrecy!**
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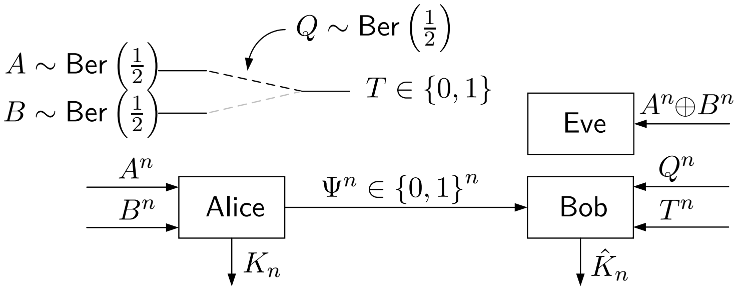
  - For $R_k = 0$ it is always beneficial to take $I(U;Y) - I(U;S) \geq 0$ (Goldfeld *et al.* 2016).

Example of the necessity of the penalty term:



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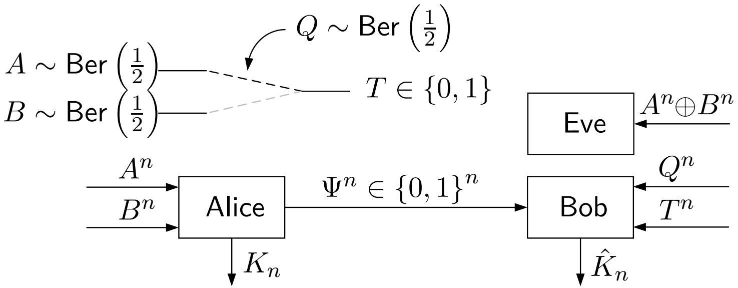

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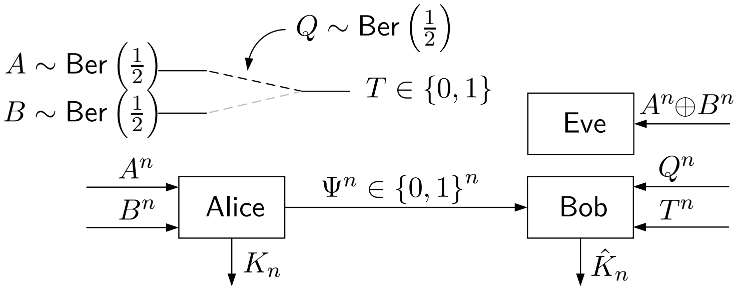

- The maximal rate of *common randomness* between Alice and Bob is 2.
- In such case $K_n \approx (A^n, B^n)$. $\implies$ The secret key rate is 1.
- $\implies$ A secret key rate of 2 is not attainable.
- However, we can choose (U, V, Ψ) such that $I(V; Y^n|U) - I(V; Z^n|U) = 2$.

- Construct a two-layered superposition codebook.

The GP WTC - Two Layer Scheme

Our results - Proof idea

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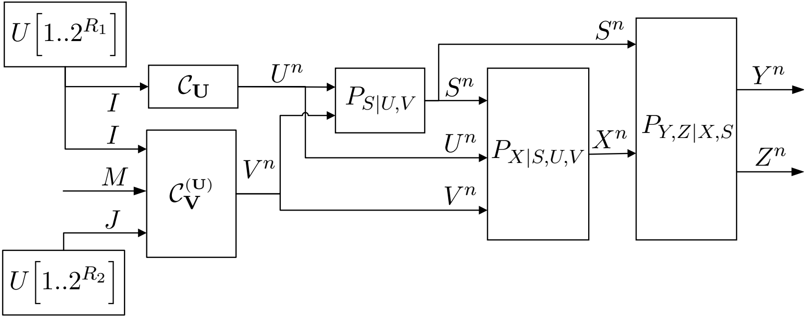

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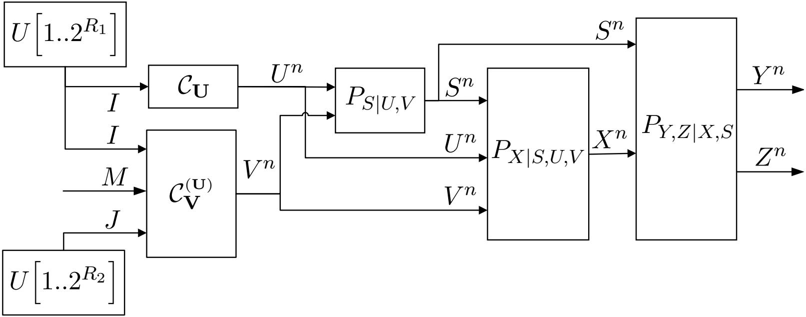

- Part of the redundancy index J may be used as a SK.

The GP WTC - Two Layer Scheme

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- Hence, the outer layer has to convey part of the inner layer index.
- However, the inner layer is decodable by the eavesdropper.
- This results in a loss of $[I(U;S) - I(U;Y)]_+$ in the secrecy rate.

The GP WTC - Two Layer Scheme

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The GP WTC - Two Layer Scheme

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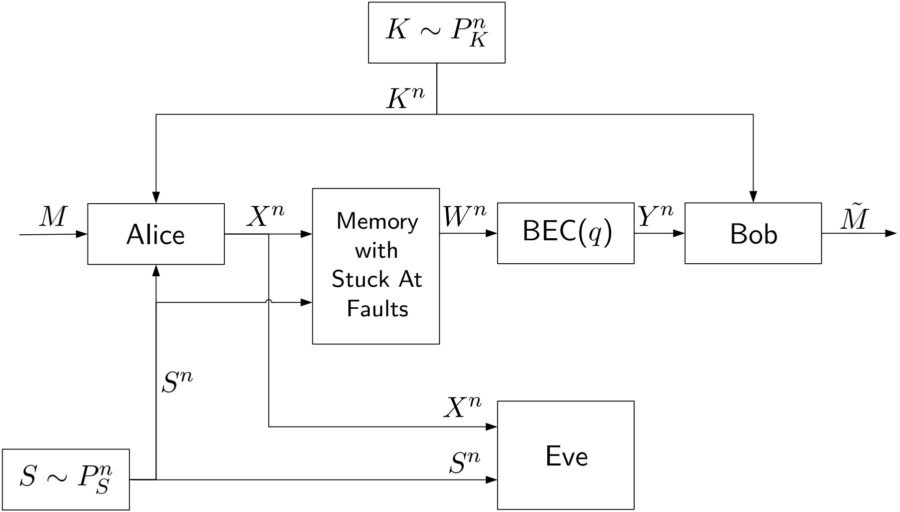
- Secrecy results for correlated sources over an independent WTC (Khisti *et al.* 2012, Bassi *et al.* 2016)

Example – Improving over Prabhakaran *et al.* 2012



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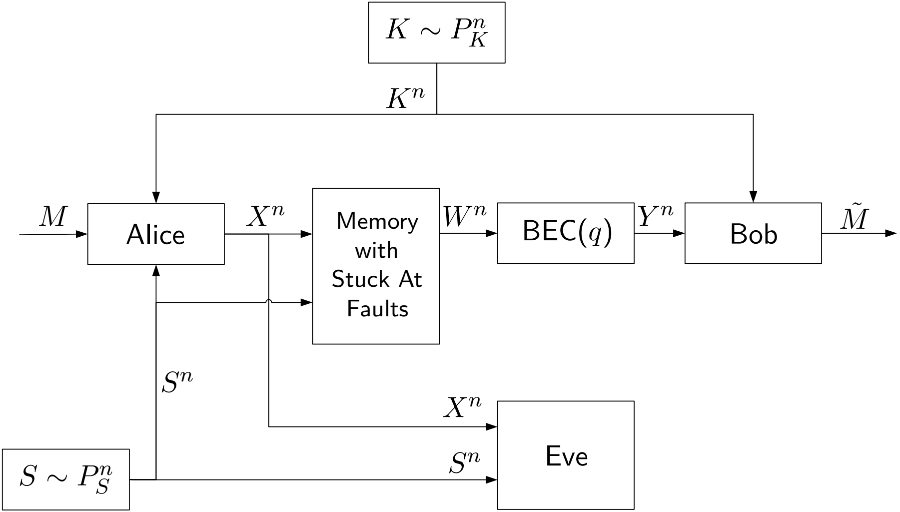

★ This example is a special case of the *state dependent less-noisy eavesdropper WTC with a key*, for which our region attains capacity.

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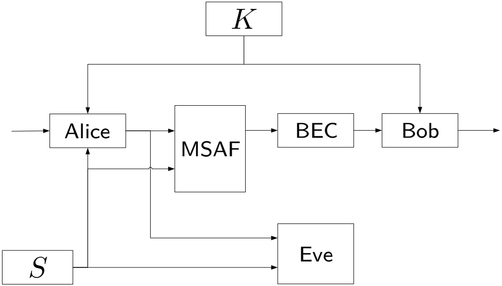

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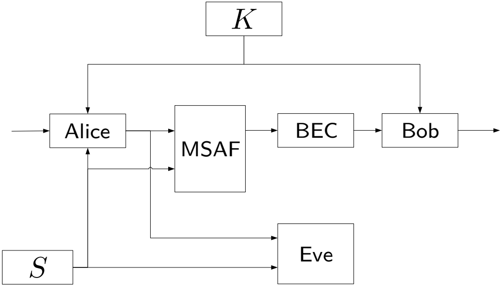

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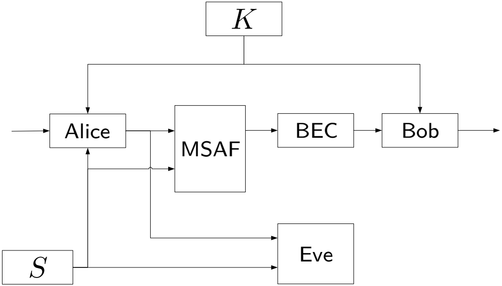

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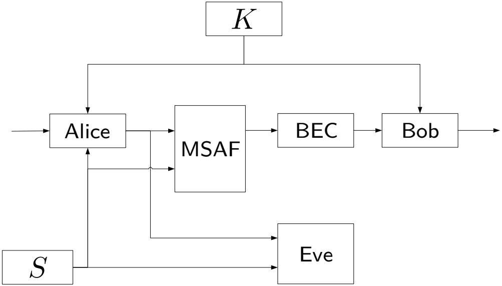

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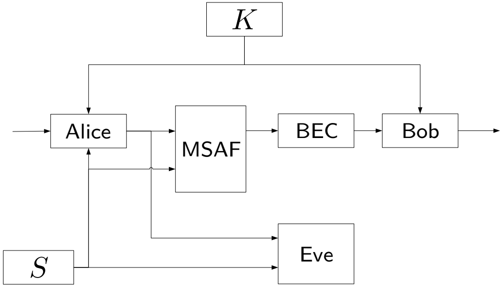

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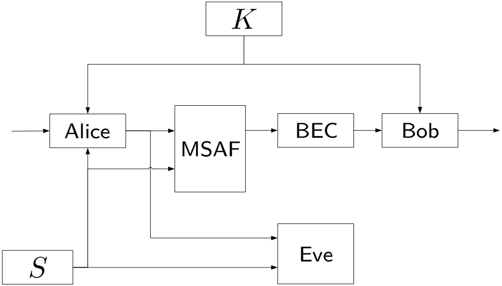

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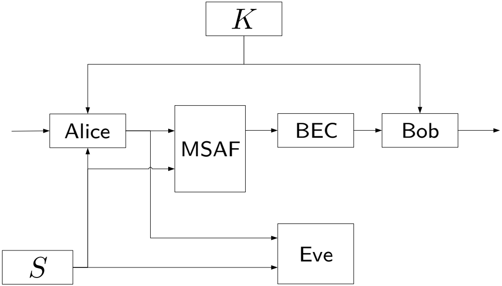

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We do!

Summary

- Several achievability results are generalized in a single scheme:
  - **Secret message** transmission over a wiretap channel
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- All these results are upgraded to **Semantic Security**.
- Strict improvement over each of the previous results. (For the general setup)

Outlook

- Outer bounds
- Multi-terminal settings
- Action dependent state
- Lossy source reconstruction over the SD-WTC
- Covert/stealthy communication
- Construction of practical codes

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Thank you!

Abstract

Two fundamental questions in physical layer security are those related to the best achievable transmission rate of a secret message (SM) over a noisy channel, and the highest attainable secret key (SK) rate that distributed parties can agree upon. We study the trade-off between SM and SK rates simultaneously achievable over a state-dependent wiretap channel with non-causal channel state information (CSI) at the encoder. This model subsumes all other instances of CSI availability as special cases, and calls for an efficient utilization of the state sequence both for reliability and security purposes.

We derive an inner bound on the SM-SK capacity region based on a novel superposition coding scheme. This inner bound improves upon the previously best known SM-SK trade-off result by Prabhakaran et al, and to the best of our knowledge, upon all other existing lower bounds for either SM or SK for this setup. The results are derived under the strict semantic-security metric that requires negligible information leakage for all message-key distributions. The achievability proof uses the likelihood encoder and the strong soft-covering lemma for superposition codes. We conclude by a short outlook.