Broadcast Approach under Information Bottleneck Capacity Uncertainty

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***Abstract*—This work considers a layered coding approach for efficient transmission of data over a wireless block fading channel, connected to a limited capacity reliable link, known as the bottleneck channel. Two main approaches are considered, the first is an oblivious approach, where the sampled noisy observations are compressed and transmitted over the bottleneck channel without having any knowledge of the original information codebook. This is compared to a decode-forward (non-oblivious) approach where the sampled noisy data is decoded, and whatever is successfully decoded is reliably transmitted over the bottleneck channel. The work is extended for an uncertain bottleneck channel capacity setting, where transmitter is not aware of the available backhaul capacity per transmission, but rather its capacity distribution. In both settings it is possible to analytically describe in closed form expressions, the optimal continuous layering power distribution which maximizes the average achievable rate.**

I. INTRODUCTION

Block fading channel model is commonly used for wireless communications, dominating the cases when mobile end-points move slow relatively to the block coherence time. In slowly varying fading channels the fading realization is fixed throughout a transmission block, giving rise to the block fading notion. By this model, the receiver can easily learn the channel characterization over the block, thus we can assume perfect Channel State Information (CSI) only at the receiver side. In most practical cases, there is no feedback channel to the transmitter, resulting in its total unawareness of the instantaneous channel, yet it knows the channel statistics.

Consider the problem of transmitting over a block fading channel to a relay node, which has to forward the received signal to a destination over a reliable link with a fixed capacity C, see Figure 1 for the schematic channel model. For Gaussian channels this is known as the bottleneck channel [1]. This channel model is also applicable for the evolving next generation 5G/6G cellular networks, where the communication with the promising architecture of the Cloud radio access network (C-RAN) introduces stringent requirements on the fronthaul capacity and latency [3], [4].

When transmitting over a block fading channel with receive CSI only, a broadcast approach [6] may be considered on the transmission to maximize the average achievable rate. The broadcast approach is essentially a variable-to-fixed channel coding approach [5]. A finite capacity link to base-station subject to random fluctuations was studied in [7] for the case of two users connecting to the same base-station. Another related overview of matrix monotonic optimization is studied in [8]. Broadcast methods for the diamond channel, which is the two parallel relays channel, were studied in [9]. Broadcast approach for bottleneck channel with a known static bottleneck capacity channel is studied in [2].

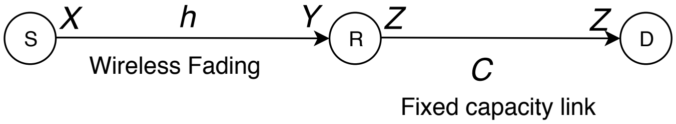

Fig. 1. Information bottleneck fading channel system model block diagram.

In the classical Gaussian bottleneck problem, depicted in Figure 1, define the random variable triplet $X-Y-Z$ forming a Markov chain, and related according to $Y = h \cdot X + N$, where X and N are independent random variables, with $N \sim N(0,1)$ being Gaussian. The signal to noise ratio (SNR) is $SNR = P|h|^2$, where the gain $|h|^2 = 1$ for a non-fading Gaussian channel, and P is the transmission power $E[X^2] = P$. The oblivious bottleneck channel output Z is a compressed version of Y adhering to a limited capacity of the bottleneck channel C. It is of interest to maximize $I(X;Z)$. Evidently if X is Gaussian it is well known by Tishby et al [1], and [10], then also $Y - Z$ is a Gaussian channel, as follows immediately from the remote rate distortion theory. The bottleneck gives reliable information rate that can be transmitted from X to Z, when the relay Y operates in an oblivious manner (it has no knowledge about the codebook and can not decode the message). For a non-oblivious approach the result is immediate, as the relay may decode the data, and then transmit over the limited capacity channel $Y - Z$ at a maximum rate C.

On a dynamic cellular network with rapidly changing loads over time, the uplink available capacity on the backhaul may be subject to high variability. Traffic congestion of internet data may lead to changing availability levels of the backhaul [7]. On the bottleneck channel this means that the relay-destination link capacity C is a random variable. It may be

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assumed that the transmitter is aware of the average capacity, and its distribution, however like in case of the wireless fading channel, the capacity variability dynamics may not allow time for feedback to the transmitter. The relay is fully aware of the bottleneck momentarily available capacity for each received codeword. This problem is considered and analyzed in this work, where the joint optimization of broadcast power allocation for multiple bottleneck capacity states is performed.

II. MAIN CONTRIBUTIONS

This work considers the problem of efficient transmission over the block fading channel with a bottleneck limited channel capacity, where bottleneck capacity uncertainty is also considered. The transmitter has no information of the fading and the available bottleneck capacity. Two main approaches are studied, the first is an oblivious approach, where the sampled noisy observations are compressed and transmitted over the bottleneck channel without having any knowledge of the original information codebook. This is compared to a non-oblivious decode-forward approach where the sampled noisy data is decoded, and whatever is successfully decoded is reliably transmitted over the bottleneck channel. The model is extended for an uncertain bottleneck channel capacity setting, where transmitter is not aware of the available backhaul capacity C per transmission, only its distribution. In both settings it is possible to analytically describe in closed form expressions, the optimal continuous layering power distribution which maximizes the average achievable rate. Fortunately, it is also possible to solve in closed form the joint optimization of the broadcast approach, under backhaul capacity uncertainty, given its distribution, for the oblivious relaying.

One of the main results here is the continuous broadcast approach [6] as optimized in closed form for the decode-forward relay, under bottleneck capacity constraint C. The transmitted signal X is multi-layer coded, in a continuum of layers. The received signal Y is decoded layer-by-layer in a successive decoding manner. All the successfully decoded layers with a total rate up to the bottleneck channel capacity C can be reliably conveyed over the bottleneck channel. The broadcast approach optimization goal is to maximize the expected transmitted rate over the bottleneck channel in this block fading channel model. We formulate here the optimization of power density distribution function $\rho_{opt}(u)$ so that average transmission rate

Proposition 1. *For the decode-forward block fading bottleneck channel, the total* expected average achievable rate of the broadcast approach *is obtained by the following residual power distribution function*

$$I_{opt}(u) = \arg\max_{I(u)} \frac{1}{2}\int_0^\infty du(1-F_\nu(u))\frac{\rho(u)u}{1+I(u)u}, \qquad (1)$$

$$s.t. \int_0^\infty du \frac{\rho(u)u}{1+I(u)u} \le C$$

where $F_\nu(u)$ is the CDF of the fading gain random variable, and C is the bottleneck channel capacity. The optimal power allocation $I_{opt}(u)$ is given by

$$I_{opt}(u) = \begin{cases} P & u < u_0 \\ \frac{1-F_\nu(u)+\lambda_{opt}-u\cdot f_\nu(u)}{u^2 f_\nu(u)} & u_0 \le u \le u_1 \\ 0 & u > u_1 \end{cases} \qquad (2)$$

where $\lambda_{opt} \ge 0$ is a Lagrange multiplier specified by

$$\lambda_{opt} = -u_1 \cdot f_\nu(u_1) - 1 + F_\nu(u_1) \qquad (3)$$

and for any $\lambda_{opt} > 0$,

$$u_1^2 \cdot f_\nu(u_1) = \exp(2C) \cdot u_0^2 \cdot f_\nu(u_0) \qquad (4)$$

Proof. The optimal residual power distribution which maximizes the rate has to maximize the average layered transmission rate under a bottleneck channel capacity limitation C. The optimization problem can be expressed as

$$R_{bs,non-Obliv,avg} = \max_{I(u)} \frac{1}{2}\int_0^\infty du(1-F_\nu(u))\frac{\rho(u)u}{1+I(u)u}, \qquad (5)$$

$$s.t. \left(C - \int_0^\infty du\frac{\rho(u)u}{1+I(u)u}\right) \ge 0$$

This constrained variational problem can be expressed with an Euler-Lagrange constrained optimization problem

$$\max_{I(u)} \frac{1}{2}\int_0^\infty du(1-F_\nu(u))\frac{\rho(u)u}{1+I(u)u} \qquad (6)$$

$$+\lambda\left(C - \int_0^\infty du\frac{\rho(u)u}{1+I(u)u}\right)$$

where $\lambda \ge 0$ is a scalar Lagrange multiplier. The problem is a standard constrained variational problem with boundary conditions. The extremum conditions of (6) are directly computed. Next, substitution of expressions in the extremum condition, and solving for $I_{opt}(u)$ gives

$$I_{opt}(u) = \frac{1-F(u)+\lambda-uf(u)}{u^2 f(u)} \qquad (7)$$

which requires applying the boundary conditions to get (2). The constant λ_{opt} is obtained from applying the boundary condition $I(u_1) = 0$, to get (3), and the total rate constraint of the bottleneck channel C is applied by

$$C = \int_{u_0}^{U_1} du\frac{-uI'_{opt}}{(1+uI_{opt}(u))} \qquad (8)$$

where I_{opt} is specified in (7). This leads to the result in (4), from

$$\exp(2C) = \frac{u_1^2 f_\nu(u_1)}{u_0^2 f_\nu(u_0)} \qquad (9)$$

and the equation in (4) is directly obtained. $\square$

Uncertainty of a bottleneck channel capacity is defined by a discrete random variable C_b, which admits to N capacity values $\{C_i\}_{i=1}^N$ with corresponding probabilities $\{p_{b,i}\}_{i=1}^N$, s.t. $p_{b,i} \geq 0$ and $\sum_{i=1}^N p_{b,i} = 1$. Under oblivious broadcasting, the combined equivalent channel viewed by the transmitter

$$FPR_{eq}(s, C_b) = \frac{s(1-\exp(-2C_b))}{1+s\cdot P\cdot \exp(-2C_b)}, \quad s = |h|^2, \quad (10)$$

Continuous broadcast approach is optimized for a fading distribution $F_\mu(u)$ where $\mu = FPR_{eq}(s, C_b)$ (10) : equivalent channel gain depending on the fading gain realization s, and bottleneck channel capacity C_b available per codeword. The cdf of this fading gain is

$$F_\mu(u) = \sum_{i=1}^N p_{b,i} F_s\left(\frac{u}{1-(1+Pu)\exp(-2C_i)}\right) \quad (11)$$

The main result here is expressed on the following proposition

Proposition 2. *The power distribution, which maximizes the expected rate over the oblivious bottleneck channel is*

$$I(x) = \begin{cases} \frac{1-F_\mu(x)-x\cdot f_\mu(x)}{x^2 f_\mu(x)} & , \; x_0 \leq x \leq x_1 \\ 0 & , \; else \end{cases} \quad (12)$$

where x_0 *is determined by* $I(x_0) = P$*, and* x_1 *by* $I(x_1) = 0$*. And the broadcasting rate is expressed as function of the* FPR_{eq} *distribution* $F_\mu(u)$

$$R_{opt}(s) = \begin{cases} 0 & s < x_0 \\ \log(s/x_0) + \frac{1}{2}\log\left(\frac{f_\mu(s)}{f_\mu(x_0)}\right) & x_0 \leq s \leq x_1 \\ \log(x_1/x_0) + \frac{1}{2}\log\left(\frac{f_\mu(x_1)}{f_\mu(x_0)}\right) & s > x_1 \end{cases} \quad (13)$$

Proof. The proof is a direct derivation of the broadcast approach optimization [6] for the power distribution under an equivalent channel model that includes the relayed signal after compression to a rate which matches the bottleneck channel capacity. The channel model for the relayed signal Z can be expressed by its block fading gain, under an oblivious approach

$$Z = \sqrt{FPR_{eq}}\cdot X + N, \quad (14)$$

where N is a unit variance Gaussian noise, and $FPR_{eq}(s, C_b)$ is specified in (10), which is directly obtained from the wireless channel model as stated in the introduction. ☐

III. AN OUTLOOK

A possible direction for further research on the information bottleneck channel is to consider a model with two relays, known as the diamond channel. In the oblivious non-fading case the optimal transmission and relay compression, together with joint decompression at the receiver are known and characterized in [12]. For the non-oblivious diamond channel only upper bounds [13] and achievable rates of the type [14] are available. Another possible direction is extending [15] to scenarios where the variable backhaul links capacities $\{C_i\}$ are not available at the relay node, but at the destination only. Successive refinement source coding techniques [17] and [18] adapted to the logarithmic loss, provide the basic tools that are in current study for this application adhering to oblivious processing.

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