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**Gaussian Diamond
Primitive Relay with
Oblivious Processing**

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GAUSSIAN DIAMOND PRIMITIVE RELAY WITH OBLIVIOUS PROCESSING

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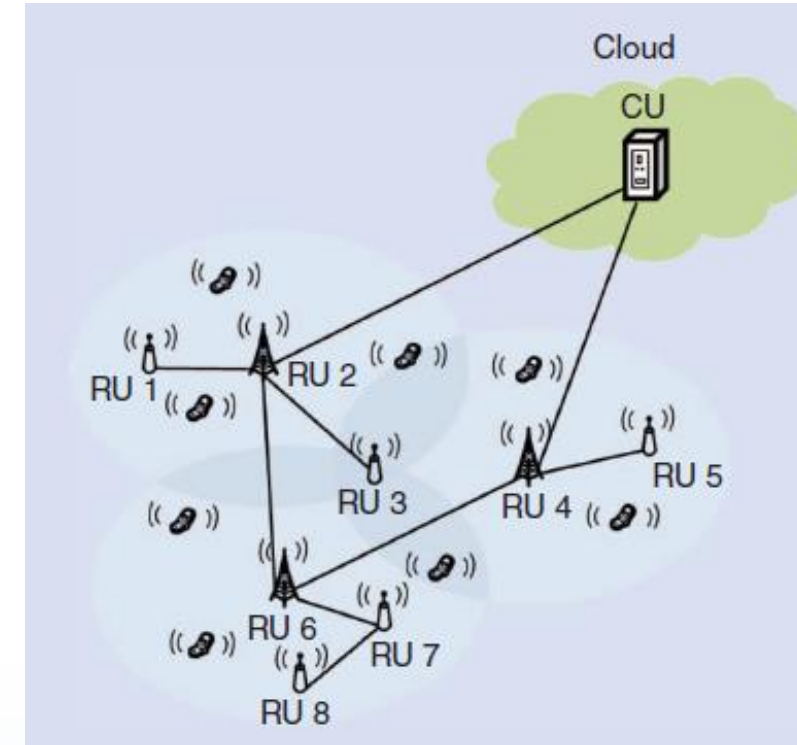
Outline

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- Information bottleneck for one relay channel communication system
- Relaying over frequency dependent channels -
Extending the water pouring approach
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- Conclusions



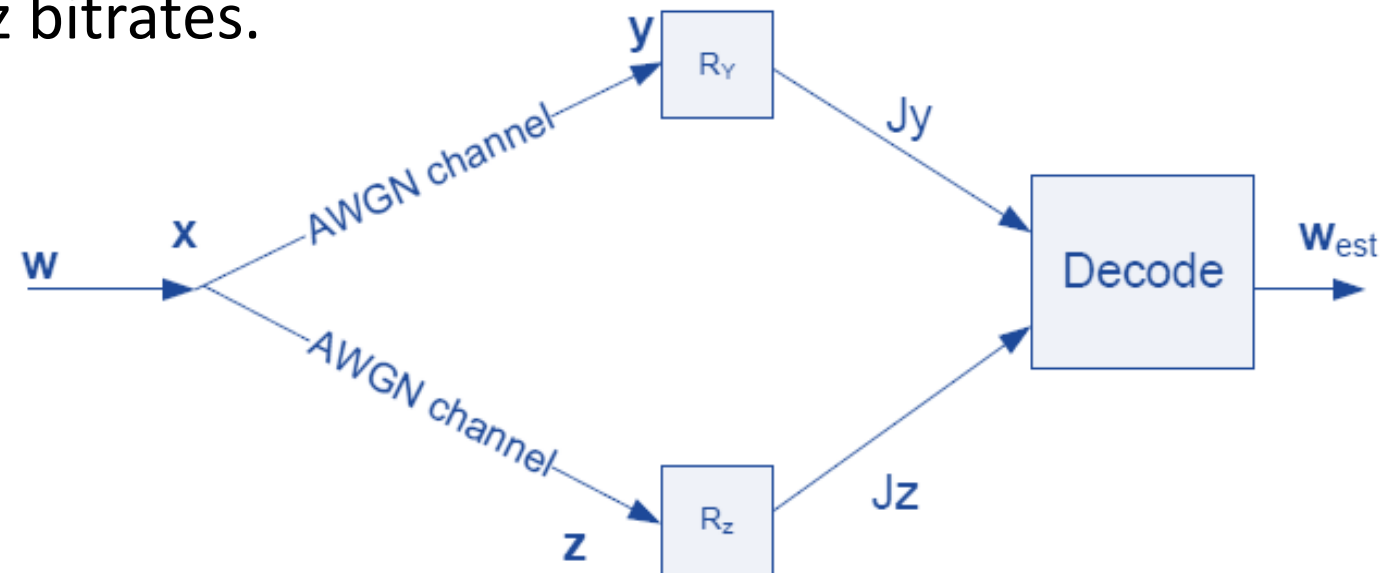
Relaying

- Relaying is used in order to improve the performance of a communication system by using intermediate nodes. In this work there is a radio channel from a transmitter to the relays and reliable bit-pipes from the relays to the destination.
- Examples are Cloud Radio Access Network (CRAN) and Remote Radio Heads (RRH) with fronthaul Common Public Radio Interface (CPRI).
- The fronthaul importance is shown in [Camps-Mur et al, 2019].
- We can distinguish between two cases: oblivious relay (such as CPRI) and non-oblivious relay.
- Oblivious relay - there is no a priori knowledge of the modulation or the coding at the relay, thus the relaying system is universal and can serve many diverse users and operators.



Diamond primitive relay channel

- We investigate the uplink using the case of Gaussian channel with identical frequency response of the channels from X to Y and Z with limited relay to destination bitrate.
- The oblivious compression and forward (CF) is used, using joint decompression/decoding (equivalent to optimized Wyner-Ziv compression).
- We use Gaussian-distributed transmission symbols which are optimal at high J_y, J_z bitrates.



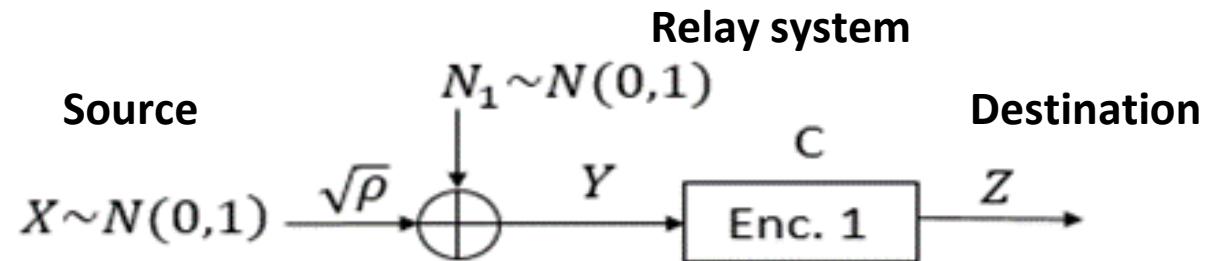
Information bottleneck for one relay channel communication system

- The optimal performance of the oblivious system with no interference, is governed by the information bottleneck method [Tishby, Pereira and Bialek, 1999] and the Gaussian Information Bottleneck (GIB).

- For the one channel case:

$$I(C) = \max_{P(z|y)} I(X; Z)$$

$$s.t. I(Y; Z) \leq C$$



- The solution for that optimization problem, was presented in [Winkelbauer and Matz, 2014]:

$$I(\rho, C) = \frac{1}{2} \log_e \left(\frac{1 + \rho}{1 + \rho e^{-2C}} \right)$$



Relaying over frequency dependent channels - Extending the water pouring approach

- Our work is about extending the known optimization over frequency-flat relay channels to more realistic frequency-dependent ones.
- For frequency dependent channel we use the water pouring approach: split the channel into separate bands, each with bandwidth of df . In the Gaussian model the different bands are independent.
- By Nyquist, for a channel with bandwidth df and no interference, the maximal symbol rate equals $2df$.
- $C(f)$ is the frequency dependent rate allocation, $S(f)$ is the frequency dependent power allocation and $H(f)$ is the filter frequency response between the source and the relay.
- For each band we assign rate of $0.5C(f)$ bits per channel use. Then the rate in this band equals $C(f)df$.
- The SNR equals $S(f)|H(f)|^2$.

One relay channel communication system

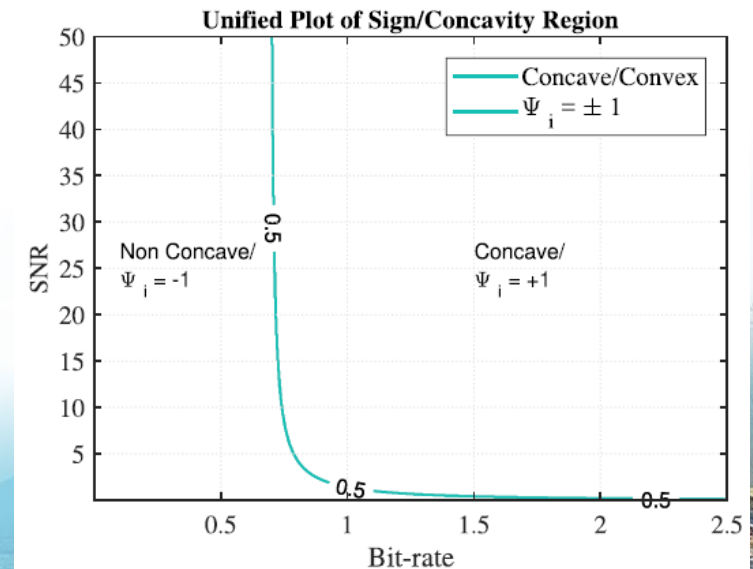
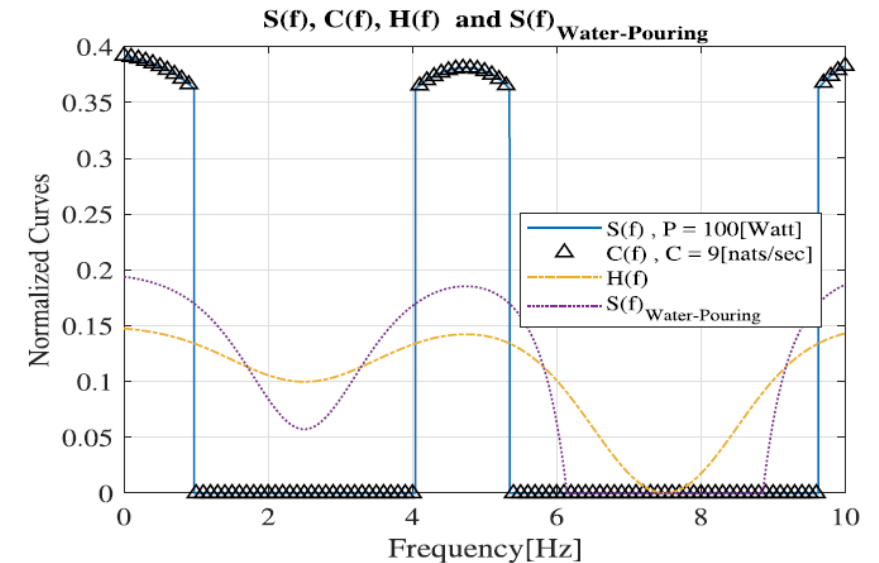
- Therefore, the frequency dependent rate is:

$$I(f, S(f), C(f)) = \log_e \left(\frac{1 + S(f) |H(f)|^2}{1 + S(f) |H(f)|^2 e^{-C(f)}} \right)$$

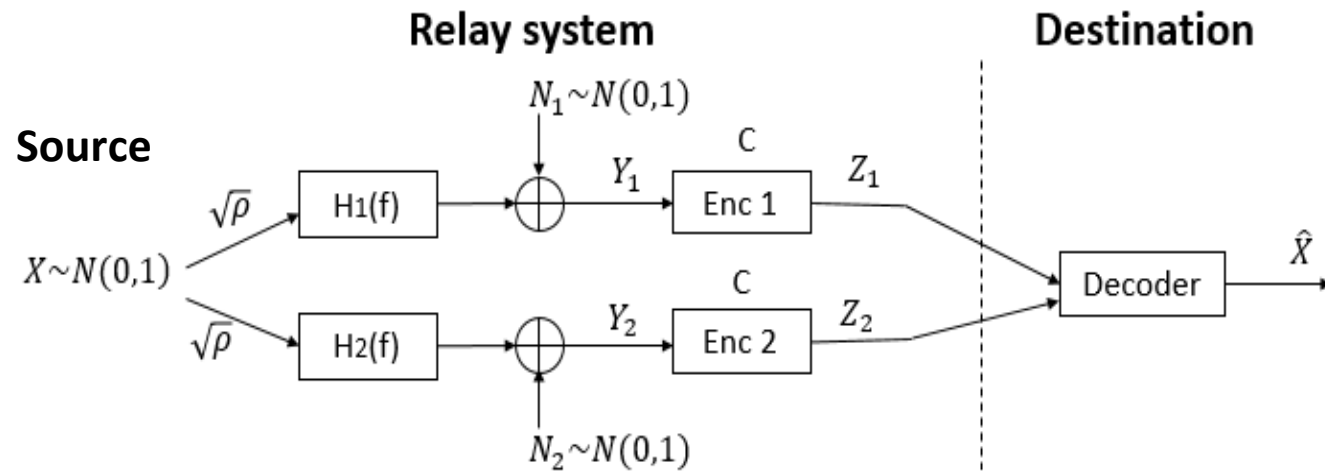
- The optimization problem becomes: $\max_{S(f), C(f)} \int_0^W I(f, S(f), C(f)) df$ s.t. $\int_0^W S(f) df \leq P, \int_0^W C(f) df \leq C$
- The LaGrangian for each frequency is: $L(f, \hat{S}, \hat{C}, \lambda_c, \lambda_s) = I(f, \hat{S}, \hat{C}) - \lambda_s \cdot \hat{S} - \lambda_c \cdot \hat{C}$
- In the region where the function is concave, find the optimal solution using the equations (Euler-Lagrange): $\nabla L = \left(\frac{dL}{dS}, \frac{dL}{dC} \right) = (0, 0)$
- The LaGrange coefficients have bounded region, so we can use grid search in order to find the optimal value.

One relay channel communication system (cont.)

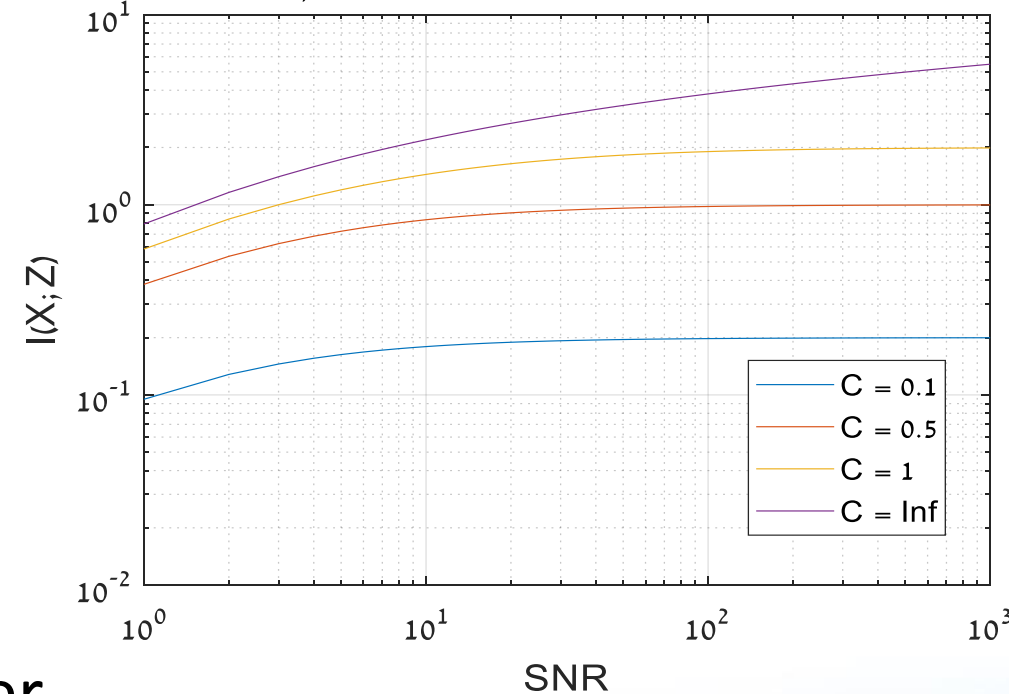
- This problem was solved in [Homri, Peleg, Shamai, 2016 and 2018].
- For each frequency we get two solutions: one in the concave region and the other in the non concave region. Choose the solution that is in the concave region, because it is the optimal one.
- The optimal solution allocates zero power and rate for certain frequencies.



Two relay channels communication system



$I(X;Z)$ vs SNR for different C rates



- Optimal solution to this problem for the discrete-time real signal case, is shown in [Sanderovich, Shamai, Steinberg and Kramer, 2008]:

$$I(\rho, C) = \frac{1}{2} \log_2 \left(1 + 2 \cdot \rho \cdot 2^{-4C} \cdot \left(2^{4C} + \rho - \sqrt{\rho^2 + (1 + 2 \cdot \rho) \cdot 2^{4C}} \right) \right)$$

Two relay channels communication system (cont.)

- Using the same water-pouring approach we get:

$$I(f, S(f), C(f)) = \log_2 \left(1 + 2 \cdot B(f) \cdot 2^{-2C(f)} \cdot \left(2^{2C(f)} + B(f) - \sqrt{A(f)} \right) \right)$$

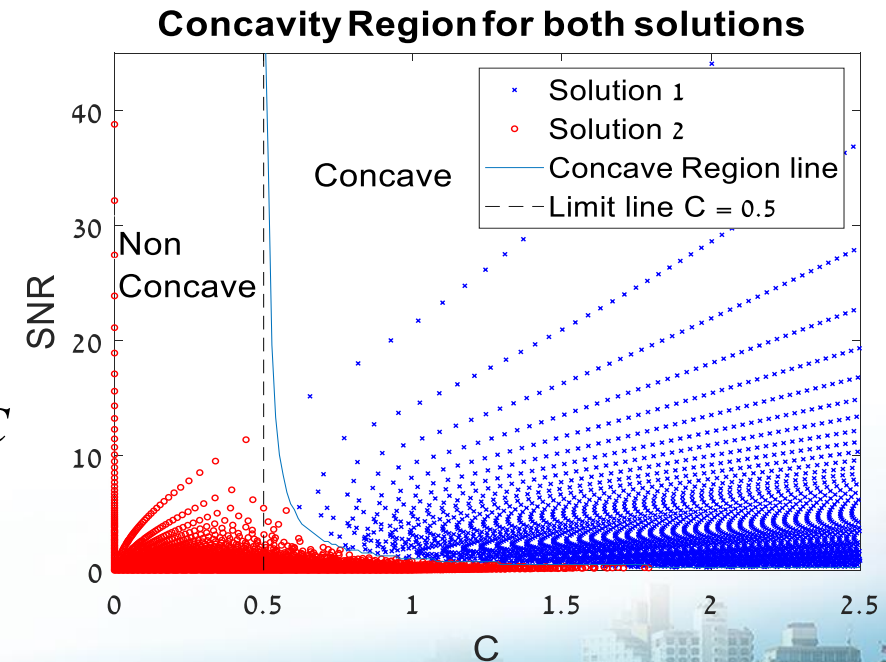
$$A(f) \triangleq (B(f))^2 + (1 + 2 \cdot B(f)) \cdot 2^{2C(f)}$$

$$B(f) \triangleq S(f) |H(f)|^2$$

- And the problem we solve in our paper is:

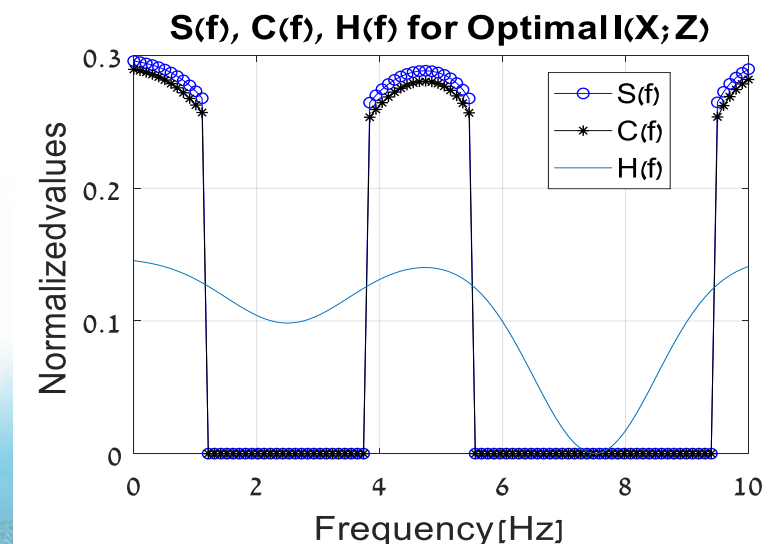
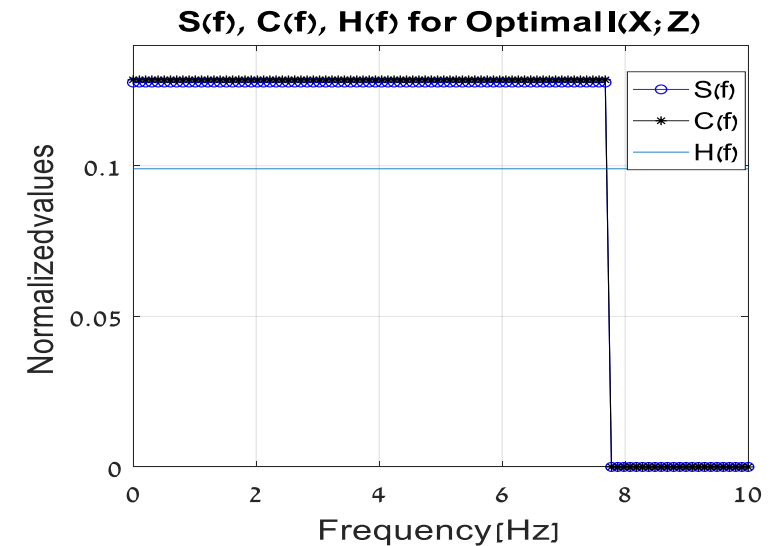
$$\max_{S(f), C(f)} \int_0^W I(f, S(f), C(f)) df \quad s.t. \quad \int_0^W S(f) df \leq P, \quad \int_0^W C(f) df \leq C$$

For the above function.



Two relay channels communication system (cont.)

- Using the same grid search method, we were able to find the optimal solution.
- Similarly, we get two solutions and choose the optimal one which is in the concave region.
- The optimal solution allocates zero power and rate for certain frequencies.



Two relay channels communication system (cont.)

- We compare the optimal solutions results for a lower and upper bounds of the capacity.
- Upper bound: cooperative encoding, the encoders can share information and operate jointly.
- Lower bound: each relay operates independently.
- The system rate results are summarized in the following table:

Case	Our optimal scheme	Collaborative encoding - upper bound	Independently encoding - lower bound	$I(X;Z)$ $C = \infty$
Frequency flat filter	15.31	16.44	12.97	43.92
Frequency dependent filter	6.45	7.51	5.85	9.55



Conclusions

- We extended known optimized relaying results over frequency flat channels to frequency-varying channels.
- We showed the advantage of applying the Joint Decompression-Decoding (optimized) Wyner-Ziv technique and investigated the loss incurred by lack of cooperation between the relays which would necessitate an additional communication link between them.
- As in [Homri, Peleg, Shamai, 2016], the optimal allocation is zero for some frequencies even over the frequency-flat channel due to the need to concentrate power and bitrate resources.

Thank you!



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