Gaussian Diamond

Primitive Relay with Oblivious Processing

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GAUSSIAN DIAMOND PRIMITIVE RELAY WITH OBLIVIOUS PROCESSING

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Outline

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Relaying

- Relaying is used in order to improve the performance of a communication system by using intermediate nodes. In this work there is a radio channel from a transmitter to the relays and reliable bit-pipes from the relays to the destination.
- Examples are Cloud Radio Access Network (CRAN) and Remote Radio Heads (RRH) with fronthaul Common Public Radio Interface (CPRI).
- The fronthaul importance is shown in [Camps-Mur et al, 2019].
- We can distinguish between two cases: oblivious relay (such as CPRI) and non-oblivious relay.
- Oblivious relay - there is no a priori knowledge of the modulation or the coding at the relay, thus the relaying system is universal and can serve many diverse users and operators.
Diamond primitive relay channel

• We investigate the uplink using the case of Gaussian channel with identical frequency response of the channels from X to Y and Z with limited relay to destination bitrate.

• The oblivious compression and forward (CF) is used, using joint decompression/decoding (equivalent to optimized Wyner-Ziv compression).

• We use Gaussian-distributed transmission symbols which are optimal at high Jy, Jz bitrates.
Information bottleneck for one relay channel communication system

• The optimal performance of the oblivious system with no interference, is governed by the information bottleneck method [Tishby, Pereira and Bialek, 1999] and the Gaussian Information Bottleneck (GIB).

• For the one channel case:

\[ I(C) = \max_{P(z|y)} I(X;Z) \]

\[ s.t \ I(Y;Z) \leq C \]

• The solution for that optimization problem, was presented in [Winkelbauer and Matz, 2014]:

\[ I(\rho, C) = \frac{1}{2} \log_e \left( \frac{1 + \rho}{1 + \rho e^{-2C}} \right) \]
Relaying over frequency dependent channels - Extending the water pouring approach

• Our work is about extending the known optimization over frequency-flat relay channels to more realistic frequency-dependent ones.

• For frequency dependent channel we use the water pouring approach: split the channel into separate bands, each with bandwidth of $df$. In the Gaussian model the different bands are independent.

• By Nyquist, for a channel with bandwidth $df$ and no interference, the maximal symbol rate equals $2df$.

• $C(f)$ is the frequency dependent rate allocation, $S(f)$ is the frequency dependent power allocation and $H(f)$ is the filter frequency response between the source and the relay.

• For each band we assign rate of $0.5C(f)$ bits per channel use. Then the rate in this band equals $C(f)df$.

• The SNR equals $S(f)|H(f)|^2$. 
One relay channel communication system

• Therefore, the frequency dependent rate is:

\[ I(f, S(f), C(f)) = \log_e \left( \frac{1 + S(f) |H(f)|^2}{1 + S(f) |H(f)|^2 e^{-C(f)}} \right) \]

• The optimization problem becomes:

\[
\max_{S(f), C(f)} \int_0^w I(f, S(f), C(f)) df \quad \text{s.t.} \quad \int_0^w S(f) df \leq P, \int_0^w C(f) df \leq C
\]

• The LaGrangian for each frequency is:

\[ L(f, \hat{S}, \hat{C}, \lambda_s, \lambda_c) = I(f, \hat{S}, \hat{C}) - \lambda_s \cdot \hat{S} - \lambda_c \cdot \hat{C} \]

• In the region where the function is concave, find the optimal solution using the equations (Euler-Lagrange):

\[ \nabla L = \left( \frac{dL}{dS}, \frac{dL}{dC} \right) = (0, 0) \]

• The LaGrange coefficients have bounded region, so we can use grid search in order to find the optimal value.
One relay channel communication system (cont.)

• This problem was solved in [Homri, Peleg, Shamai, 2016 and 2018].

• For each frequency we get two solutions: one in the concave region and the other in the non concave region. Choose the solution that is in the concave region, because it is the optimal one.

• The optimal solution allocates zero power and rate for certain frequencies.
Two relay channels communication system

- Optimal solution to this problem for the discrete-time real signal case, is shown in [Sanderovich, Shamai, Steinberg and Kramer, 2008]:

\[
I(\rho, C) = \frac{1}{2} \log_2 \left( 1 + 2 \cdot \rho \cdot 2^{-4C} \cdot \left( 2^{4C} + \rho - \sqrt{\rho^2 + (1 + 2 \cdot \rho) \cdot 2^{4C}} \right) \right)
\]
Two relay channels communication system (cont.)

- Using the same water-pouring approach we get:

\[
I(f, S(f), C(f)) = \log_2 \left( 1 + 2 \cdot B(f) \cdot 2^{-C(f)} \cdot \left( 2^{2C(f)} + B(f) - \sqrt{A(f)} \right) \right)
\]

\[
A(f) \triangleq \left( B(f) \right)^2 + (1 + 2 \cdot B(f)) \cdot 2^{2C(f)}
\]

\[
B(f) \triangleq S(f) |H(f)|^2
\]

- And the problem we solve in our paper is:

\[
\max_{S(f), C(f)} \int_0^W I(f, S(f), C(f)) df \quad s.t. \int_0^W S(f) df \leq P, \int_0^W C(f) df \leq C
\]

For the above function.
Using the same grid search method, we were able to find the optimal solution.

Similarly, we get two solutions and choose the optimal one which is in the concave region.

The optimal solution allocates zero power and rate for certain frequencies.
Two relay channels communication system (cont.)

• We compare the optimal solutions results for a lower and upper bounds of the capacity.

• Upper bound: cooperative encoding, the encoders can share information and operate jointly.

• Lower bound: each relay operates independently.

• The system rate results are summarized in the following table:

<table>
<thead>
<tr>
<th>Case</th>
<th>Our optimal scheme</th>
<th>Collaborative encoding - upper bound</th>
<th>Independently encoding - lower bound</th>
<th>( I(X;Z) ) C = ( \infty )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency flat filter</td>
<td>15.31</td>
<td>16.44</td>
<td>12.97</td>
<td>43.92</td>
</tr>
<tr>
<td>Frequency dependent filter</td>
<td>6.45</td>
<td>7.51</td>
<td>5.85</td>
<td>9.55</td>
</tr>
</tbody>
</table>
Conclusions

• We extended known optimized relaying results over frequency flat channels to frequency-varying channels.

• We showed the advantage of applying the Joint Decompression-Decoding (optimized) Wyner-Ziv technique and investigated the loss incurred by lack of cooperation between the relays which would necessitate an additional communication link between them.

• As in [Homri, Peleg, Shamai, 2016], the optimal allocation is zero for some frequencies even over the frequency-flat channel due to the need to concentrate power and bitrate resources.
Thank you!
References


• A. Winkelbauer and G. Matz, Rate information optimal gaussian channel output compression, compression,” 48 th Annual Conference on Information Sciences and Systems (CISS), no. 1-5 , August 2014

