On the Broadcast Approach over Parallel MIMO Two-state Fading Channel

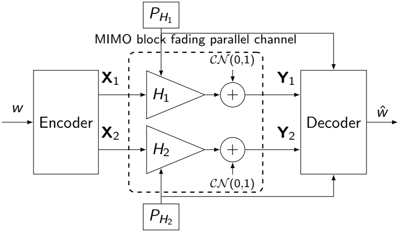
Kfir M. Cohen, Avi Steiner, Shlomo Shamai (Shitz)
Department of Electrical Engineering
Technion - Israel Institute of Technology

IZS2020, Zurich. February 2020

Outline

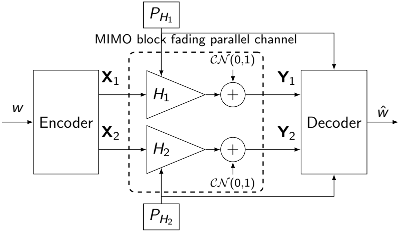
1. Channel Model
2. Background
3. Main Contribution
4. Numerical Results
5. SISO Block Fading
6. Conclusions

Channel Model (1/3)



- Single User.
- MIMO. In our case:
  - 2 TX antennas.
  - 2 RX antennas.
- Complex Parallel Channel (no cross terms).
- Block Fading (channel is coherent for N samples).
- No CSI at TX.
- Full CSI at RX.

Channel Model (2/3)



- Symmetric Channel (same statistics).
- Independent channel, across antennas and time blocks.
- Two State Channel [Kazemi,2018] [Whiting,2006], each $P_H(h) = P_A\delta(h - H_A) + P_B\delta(h - H_B)$.
  - State A: "Bad" channel (low gain)
  - State B: "Good" channel (high gain)
- $$|H_i|^2 = \begin{cases} \nu_a & \text{when } S_i = A \text{ w.p. } P_A \\ \nu_b & \text{when } S_i = B \text{ w.p. } P_B \end{cases}$$

Channel Model (3/3)

- Channel Equations per block

$$\mathbf{Y}_1 = H_1\mathbf{X}_1 + \mathbf{N}_1$$
$$\mathbf{Y}_2 = H_2\mathbf{X}_2 + \mathbf{N}_2$$

- Power Constrained $\mathbb{E}[|X_i|^2] \leq P$ on sub-channel $i \in 1, 2$.
- Shannon's Capacity considers the worst case.
- This may turn out too low.
- *Variable-to-fix coding* [Verdu,2010] allows to deliver high throughput.

Motivation

Goal of Work

Finding an encoding and decoding scheme that achieves the highest throughput.

Main Tools to Achieve

- Broadcast Approach for block fading [Shamai,1997] [Shamai,2003].
- Degraded Broadcast Product Channel [ElGamal,1980].

Broadcast Approach for Fading Channels

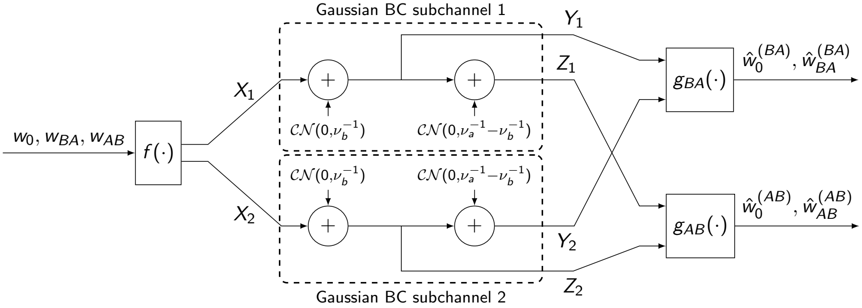
- Layering coding and successive decoding for SISO channel.
- For two layers, use power of $\alpha^{\text{bc}}P$ to overcome the bad channel while treating second layer as noise, and $(1-\alpha^{\text{bc}})P$ for the second layer.
- Average throughput (two layers)

$$R_{\text{avg}}^{\text{bc}}(\alpha^{\text{bc}}) = P_A \log\left(1+\frac{\nu_a\alpha^{\text{bc}}P}{1+\nu_a(1-\alpha^{\text{bc}})P}\right) + \text{P}_B\left[\log\left(1+\frac{\nu_a\alpha^{\text{bc}}P}{1+\nu_a(1-\alpha^{\text{bc}})P}\right)+\log\left(1+\nu_b(1-\alpha^{\text{bc}})P\right)\right].$$

- Maximal average throughput by

$$\alpha^{\text{bc,opt}} = \max\left\{0, \min\left\{1, 1-\frac{P_B\nu_b-(P_A+P_B)\nu_a}{P_A\nu_a\nu_b P}\right\}\right\}.$$

El-Gamal 1980 Degraded Broadcast Product Channels



$$R_0 \le \log\left(1+\frac{\nu_a\alpha P}{1+\nu_a\bar{\alpha}P}\right)+\log\left(1+\frac{\nu_b\alpha P}{1+\nu_b\bar{\alpha}P}\right)$$

$$R_0+R_{BA}=R_0+R_{AB}\le \log\left(1+\frac{\nu_a\alpha P}{1+\nu_a\bar{\alpha}P}\right)+\log\left(1+\nu_b P\right)$$

$$R_0+R_{BA}+R_{AB}\le \log\left(1+\nu_b P\right)+\log\left(1+\frac{\nu_a\alpha P}{1+\nu_a\bar{\alpha}P}\right)+\log\left(1+\nu_b\bar{\alpha}P\right).$$

Main Contribution: Adding states AA and BB to El-Gamal

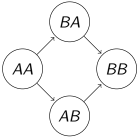
In red: El Gamal

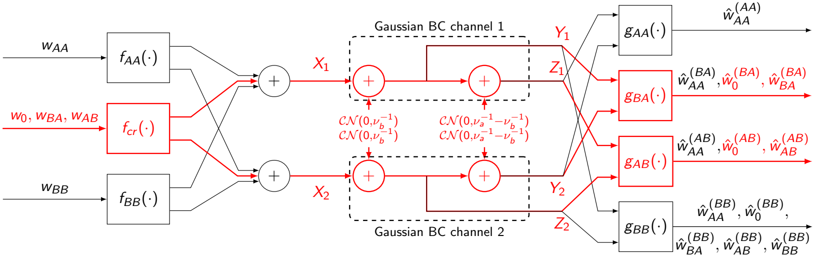
In Black: Additional states

AA: bad-bad states

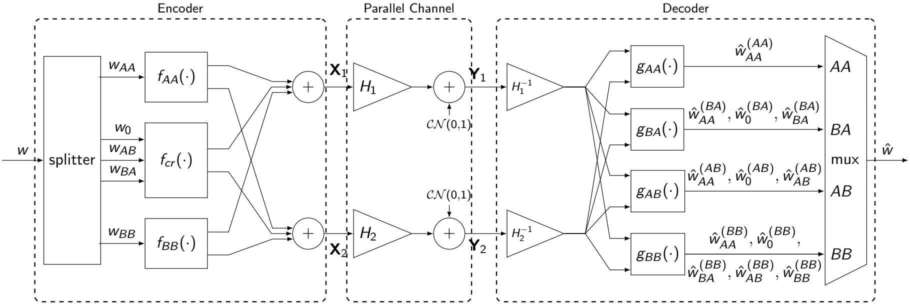
BB: good-good states

cr: bad-good or good-bad crossed states





Main Contribution: Applying on the channel model



Main Contribution: Encoding procedure

Stage no.	codewords	Power	Interference and Noise Power
1	w_{AA}	$\alpha_{AA}P$	$\nu_a^{-1} + (\alpha_{cr} + \alpha_{BB})P$
2	w_0, w_{AB}, w_{BA}	$\alpha_{cr}P$	$\nu_a^{-1} + \alpha_{BB}P$; and $\nu_b^{-1} + \alpha_{BB}P$
3	w_{BB}	$\alpha_{BB}P$	ν_b^{-1}

S	$\hat{w}_{AA}$	$\hat{w}_0$	$\hat{w}_{BA}$	$\hat{w}_{AB}$	$\hat{w}_{BB}$	Sum Rate	Probability
(A, A)	$\hat{w}_{AA}^{(AA)}$	-	-	-	-	R_{AA}	P_A^2
(A, B)	$\hat{w}_{AA}^{(AB)}$	$\hat{w}_0^{(AB)}$	-	$\hat{w}_{AB}^{(AB)}$	-	$R_{AA} + R_0 + R_{AB}$	$P_A P_B$
(B, A)	$\hat{w}_{AA}^{(BA)}$	$\hat{w}_0^{(BA)}$	$\hat{w}_{BA}^{(BA)}$	-	-	$R_{AA} + R_0 + R_{BA}$	$P_B P_A$
(B, B)	$\hat{w}_{AA}^{(BB)}$	$\hat{w}_0^{(BB)}$	$\hat{w}_{BA}^{(BB)}$	$\hat{w}_{AB}^{(BB)}$	$\hat{w}_{BB}^{(BB)}$	$R_{AA} + R_0 + R_{AB} + R_{BA} + R_{BB}$	P_B^2

Main Contribution: Defining the sum rate

S_1/S_2	A	B
A	R_{AA}	$R_{AA}+R_0+R_{AB}$
B	$R_{AA}+R_0+R_{BA}$	$R_{AA}+R_0+R_{AB}+R_{BA}+R_{BB}$

$$\begin{aligned} R_{\text{avg}} &= P_A^2 R_{AA} + P_A P_B (R_{AA}+R_0+R_{AB}) \\ &\quad + P_B P_A (R_{AA}+R_0+R_{BA}) \\ &\quad + P_B^2 (R_{AA}+R_0+R_{BA}+R_{AB}+R_{BB}) \end{aligned}$$

Main Contribution: Rates under Broadcast Approach

$$
\begin{aligned}
R_{AA} &\leq 2\log\left(1+\frac{\alpha_{AA}P}{\nu_a^{-1}+\bar{\alpha}_{AA}P}\right)\\
R_{AA}+R_0 &\leq 2\log\left(1+\frac{\alpha_{AA}P}{\nu_a^{-1}+\bar{\alpha}_{AA}P}\right)+\log\left(1+\frac{\alpha\alpha_{cr}P}{\nu_b^{-1}+(\bar{\alpha}\alpha_{cr}+\alpha_{BB})P}\right)\\
&\quad+\log\left(1+\frac{\alpha\alpha_{cr}P}{\nu_a^{-1}+(\bar{\alpha}\alpha_{cr}+\alpha_{BB})P}\right)\\
R_{AA}+R_0+R_{BA} &= R_{AA}+R_0+R_{AB}\\
&\leq 2\log\left(1+\frac{\alpha_{AA}P}{\nu_a^{-1}+\bar{\alpha}_{AA}P}\right)+\log\left(1+\frac{\alpha\alpha_{cr}P}{\nu_a^{-1}+(\bar{\alpha}\alpha_{cr}+\alpha_{BB})P}\right)\\
&\quad+\log\left(1+\frac{\alpha_{cr}P}{\nu_b^{-1}+\alpha_{BB}P}\right)\\
R_{AA}+R_0+R_{BA}+R_{AB} &\leq 2\log\left(1+\frac{\alpha_{AA}P}{\nu_a^{-1}+\bar{\alpha}_{AA}P}\right)+\log\left(1+\frac{\alpha_{cr}P}{\nu_b^{-1}+\alpha_{BB}P}\right)\\
&\quad+\log\left(1+\frac{\alpha\alpha_{cr}P}{\nu_a^{-1}+(\bar{\alpha}\alpha_{cr}+\alpha_{BB})P}\right)+\log\left(1+\frac{\bar{\alpha}\alpha_{cr}P}{\nu_b^{-1}+\alpha_{BB}P}\right)\\
R_{AA}+R_0+R_{BA}+R_{AB}+R_{BB} &\leq 2\log\left(1+\frac{\alpha_{AA}P}{\nu_a^{-1}+\bar{\alpha}_{AA}P}\right)+\log\left(1+\frac{\alpha_{cr}P}{\nu_b^{-1}+\alpha_{BB}P}\right)\\
&\quad+\log\left(1+\frac{\alpha\alpha_{cr}P}{\nu_a^{-1}+(\bar{\alpha}\alpha_{cr}+\alpha_{BB})P}\right)+\log\left(1+\frac{\bar{\alpha}\alpha_{cr}P}{\nu_b^{-1}+\alpha_{BB}P}\right)\\
&\quad+2\log\left(1+\frac{\alpha_{BB}P}{\nu_b^{-1}}\right)
\end{aligned}
$$

(1)

Main Contribution: Optimization Formulation

$$
\begin{aligned}
R_{\text{avg}} \quad = \quad & P_A^2 \left[2 \log \left(1 + \frac{\alpha_{AA} P}{\nu_a^{-1} + \bar{\alpha}_{AA} P} \right) \right] + 2 P_A P_B \left[2 \log \left(1 + \frac{\alpha_{AA} P}{\nu_a^{-1} + \bar{\alpha}_{AA} P} \right) \right. \\
& \left. + \log \left(1 + \frac{\alpha \alpha_{cr} P}{\nu_a^{-1} + (\bar{\alpha} \alpha_{cr} + \alpha_{BB}) P} \right) + \log \left(1 + \frac{\alpha_{cr} P}{\nu_b^{-1} + \alpha_{BB} P} \right) \right] \\
& + P_B^2 \left[2 \log \left(1 + \frac{\alpha_{AA} P}{\nu_a^{-1} + \bar{\alpha}_{AA} P} \right) + \log \left(1 + \frac{\alpha_{cr} P}{\nu_b^{-1} + \alpha_{BB} P} \right) \right. \\
& + \log \left(1 + \frac{\alpha \alpha_{cr} P}{\nu_a^{-1} + (\bar{\alpha} \alpha_{cr} + \alpha_{BB}) P} \right) + \log \left(1 + \frac{\bar{\alpha} \alpha_{cr} P}{\nu_b^{-1} + \alpha_{BB} P} \right) \\
& \left. + 2 \log \left(1 + \frac{\alpha_{BB} P}{\nu_b^{-1}} \right) \right].
\end{aligned}
$$

should be optimized over the 4 dimensional power allocation vector

$$
D' = \left\{ \boldsymbol{\alpha}' = [\alpha, \alpha_{AA}, \alpha_{cr}, \alpha_{BB}]^T \in \mathbb{R}^4 \;\middle|\; \begin{array}{l} \alpha_{AA} \geq 0, \alpha_{cr} \geq 0, \alpha_{BB} \geq 0 \\ 0 \leq \alpha \leq 1, \alpha_{AA} + \alpha_{cr} + \alpha_{BB} = 1 \end{array} \right\}
$$

Main Contribution: Optimizing the sum rate

The optimization problem of R_{avg} over the 4 dimensional can be reduced to a single dimensional problem.

Main Theorem

The maximal sum rate of the symmetric two parallel two state channel over all power allocations $\boldsymbol{\alpha}' = [\alpha, \alpha_{AA}, \alpha_{cr}, \alpha_{BB}]^T \in D'$ is

$$\begin{aligned} R_{\text{avg}}^{\text{opt}} &= 2(P_A + P_B)^2 \log(1 + \nu_a P) \\ &+ \max_{0 \le \alpha_{AA} \le 1} \left\{ R_0(1 - \alpha_{AA}) + R_1((1 - \alpha_{AA}) \cdot (1 - \alpha^{\text{opt}}(\alpha_{AA}))) \right\} \end{aligned}$$

where

$$\alpha^{\text{opt}}(\alpha_{AA}) = \max \left\{ 0, \min \left\{ 1, 1 - \frac{P_B^2 \nu_b - [(P_A + P_B)^2 - P_A^2]\nu_a}{2P_A \cdot P_B \cdot \nu_a \nu_b P(1 - \alpha_{AA})} \right\} \right\}.$$

$$\begin{aligned} R_0(\alpha_0) &= [(P_A + P_B)^2 - P_A^2] \log(1 + \nu_b \alpha_0 P) - [(P_A + P_B)^2 + P_A^2] \log(1 + \nu_a \alpha_0 P) \\ R_1(\alpha_1) &= P_B^2 \log(1 + \nu_b \alpha_1 P) - [(P_A + P_B)^2 - P_A^2] \log(1 + \nu_a \alpha_1 P). \end{aligned}$$

Main Contribution: Corollaries

Corollary I

$\alpha_{BB}^{\text{opt}} = 0$.
True for any set of parameters ν_a, ν_b, P_A, P_B, even if $P_B \to 1$ and $\nu_b \gg \nu_a$.

Corollary II

Under the optimal power allocation,
$\alpha^{\text{opt}}(\alpha_{AA}) = 1 - \alpha_1^{\text{opt}}(\alpha_{AA})/(1 - \alpha_{AA})$.

Defining Sub-Optimal Schemes for Comparison

Independent Broadcasting

Disjointly encoding different messages into each single channel using the broadcast approach.

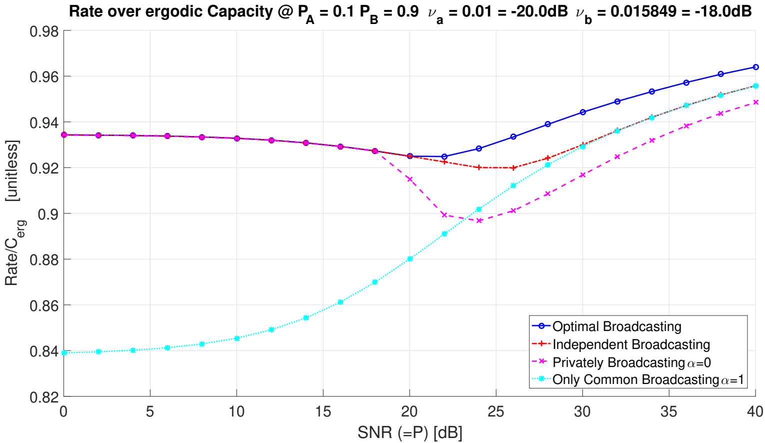
Privately Broadcasting

Allocating no power to the common stream of the crossed states (w_0). Special case by setting $\alpha = 0$.

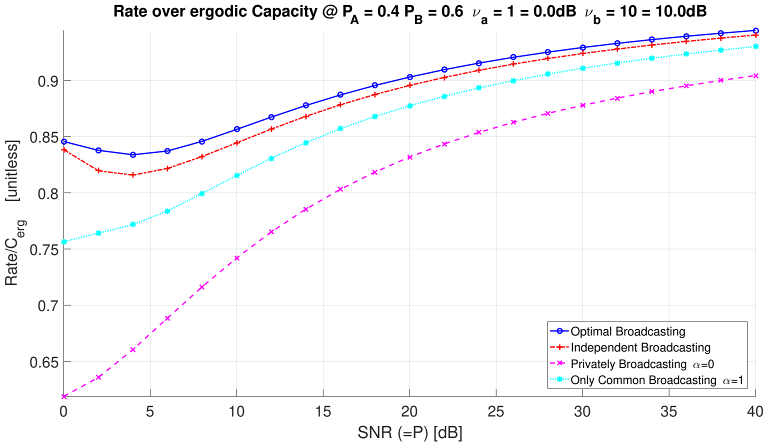
Only Common Broadcasting

Allocating no power to the private streams of the crossed states (w_{AB} and w_{BA}). Special case by setting $\alpha = 1$.

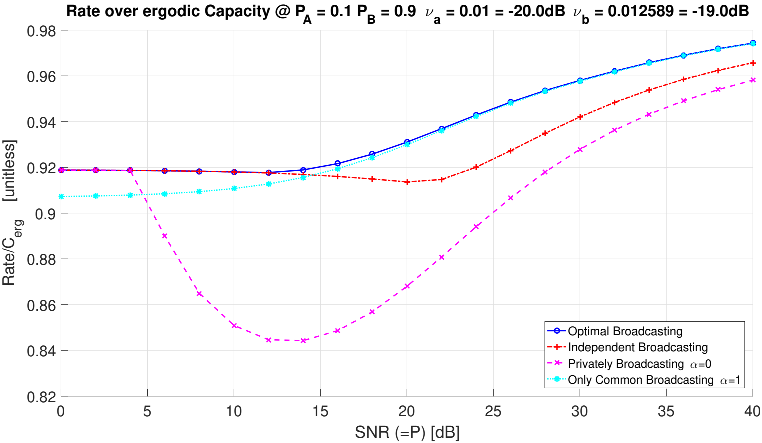
Numerical Results 1/4



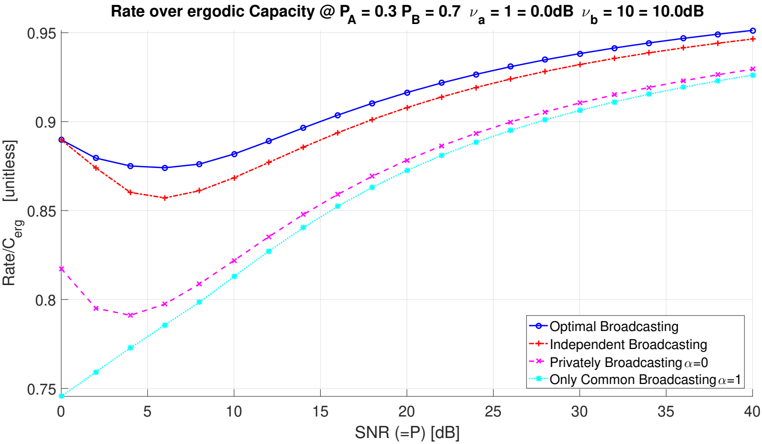
Numerical Results 2/4



Numerical Results 3/4

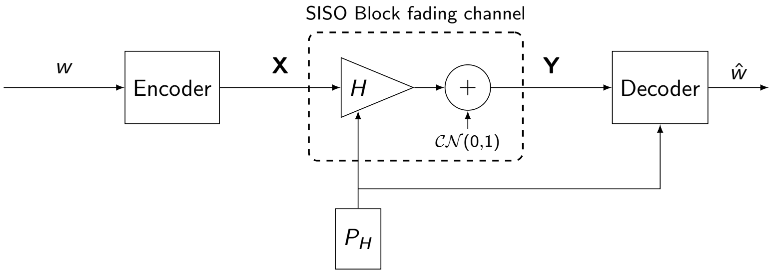


Numerical Results 4/4



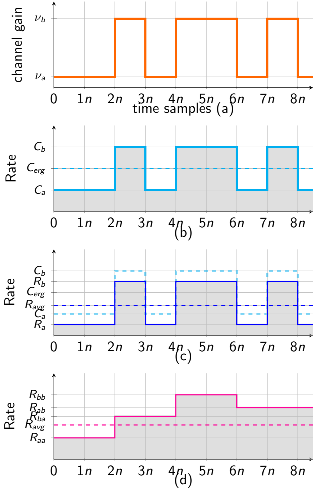
SISO Block Fading Channel 1/2

This model fits also the SISO block fading channel encoding jointly two consecutive blocks.



This channel and coding for the two state channel was introduced by [Whiting,2006].

SISO Block Fading Channel 2/2



A comment on [Whiting,2006]

- It is a special case When $\alpha = 1$.
- Optimal α is not 1, analytically proven in this work.
- They consider $\alpha_{BB} = 0$ without justification. In this work it has been analytically proven to be optimal.

Conclusion

- Applied broadcast approach on the two state parallel channel.
- Used El-Gamal channel for the common and private messages for the crossed states.
- Optimization order reduced to single variable.
- This scheme is superior over the disjointly encoders.
- Applicable over the SISO fading as well.

Thank You!

References

1. [Shamai,1997] S. Shamai (Shitz), A Broadcast Strategy for the Gaussian Slowly Fading Channel, IEEE Int. Symp. Inform. Theory (ISIT97), p. 150, Ulm, Germany, June 29-July 1, 1997.
2. [ElGamal,1980] A. El Gamal, "Capacity of the product and sum of two unmatchedbroadcast channels,"Prob. Pere. Inf., vol. 16, no. 1, pp. 1–16, Jan.-March 1980 (English Translation).
3. [Shamai,2003] S. Shamai (Shitz) and A. Steiner, "A broadcast approach for a single user slowly fading MIMO channel,"IEEE Trans. on Inf. The., vol. 49,no. 10, pp. 2617–2635, Oct. 2003.
4. [Whiting,2006] . A. Whiting and E. M. Yeh, "Broadcasting over uncertain channelswith decoding delay constraints,"IEEE Trans. Info. The., vol. 52, no. 3,pp. 904–921, March 2006.
5. [Verdu,2010] S. Verdu and S. Shamai, "Variable-rate channel capacity,"IEEE Trans.on Inf. The., vol. 56, no. 6, pp. 2651–2667, June 2010.
6. [Kazemi,2018] S. Kazemi and A. Tajer, "Multiaccess communication via a broadcastapproach adapted to the multiuser channel,"IEEE Trans. on. Comm.,vol. 66, no. 8, pp. 3341–3353, Aug 2018