International Zurich Seminar on Information and Communication (IZS), February 26 – 28, 2020

On the Broadcast Approach over Parallel MIMO Two-state Fading Channel

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***Abstract*—The single user parallel multiple input multiple output (MIMO) slow (block) flat fading channel, subject to a two-state fading per channel with additive white Gaussian noise (AWGN) is examined. The fading in each of the parallel channels is interpreted as state, which takes on two values with prescribed probabilities. We focus here on the variable to fixed channel rate (the broadcast approach) where a novel view of extension of El-Gamal's capacity of degraded broadcast product channels is examined. The optimized average rate is analytically derived over the parameters of the proposed scheme, and comparison to the simple scheme that employs the broadcast approach per each of the parallel channels separately. The achievable improvement in rates under the latency demand (transmission in a single fading block) is reflected.**

I. Introduction

Recent growth in bandwidth requirements of the 5G wireless communications networks, under stringent low latency requirements lead to vast contributions of innovations. This work focuses on the slow (block) fading parallel MIMO channel [1], where channel state is known at the receiver only. Under this channel model the transmitter may adopt a broadcast approach [2], which can optimize the expected transmission rate under no transmission channel state information (CSI), which is essentially characterized by the *variable-to-fixed coding* [3].

The *broadcast approach* [2] for slow flat-fading channels [4] uses the degradedness nature of the fading channel and applies multi-layer coding, to deliver variable-to-fixed coding over block fading channels. The amount of successfully decoded layers depends on the channel realization. For deeply fading channels few layers are decoded, while for high fading gains, more layers can be decoded. Rate and power allocation per layer are optimized to maximize the expected rate. The broadcast approach can be compared to the ergodic bound [5], achievable given transmit CSI, and other contributions such as [6]–[14].

El-Gamal [15] composed two degraded broadcast channels [16], [17] into a three-user setup: an encoder with two outputs, each driving a dual-output broadcast channel; two decoders, each is input by one less-noisy broadcast channel output and one more-noisy output of the other channel (called 'unmatched'). This was coined *degraded broadcast product channel*. For the AWGN case, the capacity region (private and common rates) was derived.

In this paper, the MIMO setup for the broadcast approach is revisited, with new tools that differ from those in [2], [18]. This is by analyzing the finite state parallel MIMO channel, where El-Gamal's capacity region [15] is used to address the multi-layering optimization problem for maximizing the expected rate of a two-state fading [19]–[21] parallel MIMO channel.

II. Channel Model

Consider a single user parallel MIMO channel setting, where a message w is to be block-encoded and sent through a diagonal matrix two-input two-output flat fading channel depicted in Fig. 1. The channel is given by

$$\begin{aligned} \mathbf{Y}_1 &= H_1\mathbf{X}_1 + \mathbf{N}_1, \\ \mathbf{Y}_2 &= H_2\mathbf{X}_2 + \mathbf{N}_2, \end{aligned} \tag{1}$$

where $\mathbf{Y}_i \in \mathbb{C}^n$ is the received n-length symbols vector on channel $i \in 1,2$, $\mathbf{X}_i \in \mathbb{C}^n$ is the transmitted vector over channel i which satisfies the power constraint $\mathbb{E}[|X_i|^2] \le P$, $i \in 1,2$. The additive noise vector is denoted $\mathbf{N}_i \in \mathbb{C}^n$ and its elements are complex normal i.i.d with zero mean and unit variance $\mathcal{CN}(0,1)$. The i-th sub-channel fading coefficient is denoted $H_i \in \mathbb{R}_+$, is drawn by some probability function $P_H(\cdot)$ and its value remains fixed during a block transmission, changes along blocks independently, and H_1 and H_2 are statistically independent. These channel states are known only to the receiver side and are not fed back to the transmitter. With no loss of generality, the channel fading H_i is assumed to be real and positive.

For a given realization set of channel states $\{H_1, H_2\}$ known to both the transmitter and receiver, the per-block Shannon capacity is well known [1]. Since H_1 and H_2 are unknown to the transmitter, setting the rates to withstand the worst (lowest) possible H_i may occur a great deal of rate loss. Variable-to-fixed coding allows to deliver higher throughput, at the expense that only parts of the message are decodable, according the channel conditions. Clearly, the expected achievable rate can be higher than the worst-case classical capacity. The recovered message $\hat{w}$ has different cardinality upon the realization set.

In this work, the channel model is limited to a two-state symmetric case. Each channel $i = 1,2$ can have independent fading gain realizations $S_i \in \{A, B\}$, state A denotes a fading coefficient $H_i = H_A$ with probability P_A; whereas state B refers to the sub-channel $H_i = H_B$, and $|H_A| < |H_B|$, and is with probability $P_B = 1 - P_A$. This is reflected by the condition $P_H(h) = P_A\delta(h - H_A) + P_B\delta(h - H_B)$ where $\delta(\cdot)$ is the kronecker delta.For brevity, denote the fading gains by $\nu = |H|^2$, $\nu_a = |H_A|^2$ and $\nu_b = |H_B|^2$ and by definition $\nu_b > \nu_a$. The common power constraint is given by $\mathbb{E}[|X_i|^2] \le P$, $i = 1,2$. The ergodic capacity of the two state fading parallel MIMO channel is specified by $C_{\text{erg}} = 2(P_A \log(1 + P\nu_a) + P_B \log(1 + P\nu_b))$.

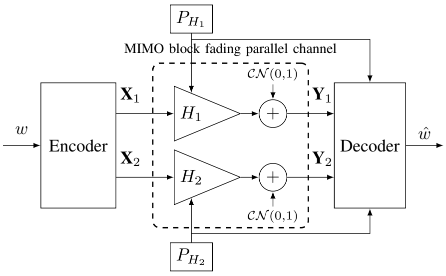

Fig. 1: The parallel MIMO block fading channel with channel state information at receiver. All codewords are of length n.

III. Preliminary: Capacity of Degraded Gaussian Broadcast Product Channels

Consider the model introduced in [15]: two receiver discrete memoryless degraded product broadcast channels. The Gaussian case was addressed as a special case. A single transmitter codes two n-length codewords consisting of a common message $w_0 \in \{1, ..., 2^{nR_0}\}$ to be decoded by both users, and two private messages $w_{BA} \in \{1, ..., 2^{nR_{BA}}\}$ and $w_{AB} \in \{1, ..., 2^{nR_{AB}}\}$, one for each of the two decoding users. A single function codes these 3 messages into two codewords; each undergoes parallel degraded broadcast subchannels

$$\begin{cases} Y_1 = X_1 + N_{11} \\ Z_1 = Y_1 + N_{12} \end{cases} \qquad \begin{cases} Z_2 = X_2 + N_{21} \\ Y_2 = Z_2 + N_{22}, \end{cases}$$

and $N_{11}, N_{21} \sim \mathcal{CN}(0, \nu_b^{-1})$, $N_{21}, N_{22} \sim \mathcal{CN}(0, \nu_a^{-1} - \nu_b^{-1})$.

As depicted in the bold and red parts of Fig. 2, two users (namely AB and BA) receive both common and private messages from the transmitter independently decode the messages. This is an unmatched setting, as Y_1 is less noisy than Z_1, alas Z_2 is less noisy than Y_2. Hence, each of the users has one less-noisy channel output alongside another which is the noisier output of the other sub-channel.

Following Theorem 2 of [15] which shows this case, and exploiting symmetry for equal power allocation to both sub-channels, optimal allocation is expected to be achieved by equal common rate allocation to every user (state). Denoting $\bar{\alpha} = 1 - \alpha$, the capacity region (R_0, R_{BA}, R_{AB}) is

$$\begin{aligned} R_0 &\le \log\left(1 + \tfrac{\nu_a \alpha P}{1+\nu_a \bar{\alpha} P}\right) + \log\left(1 + \tfrac{\nu_b \alpha P}{1+\nu_b \bar{\alpha} P}\right) \\ R_0 + R_{BA} &= R_0 + R_{AB} \le \log\left(1 + \tfrac{\nu_a \alpha P}{1+\nu_a \bar{\alpha} P}\right) + \log(1 + \nu_b P) \\ R_0 + R_{BA} + R_{AB} &\le \log(1 + \nu_b P) + \log\left(1 + \tfrac{\nu_a \alpha P}{1+\nu_a \bar{\alpha} P}\right) \\ &\qquad + \log(1 + \nu_b \bar{\alpha} P). \end{aligned} \tag{2}$$

IV. Main Contribution

A. Extended Degraded Gaussian Broadcast Product Channels

The classical product channel is extended by introducing two dual-input receivers in addition to the original two. The first has the two more noisy channel outputs (Z_1, Y_2), whereas the second gets the two less noisy outputs (Z_2, Y_1). To support this, two messages w_{AA} and w_{BB} are added. The total two n-length codewords are the superposition of three codewords by independent encoders as follows $(\mathbf{X}_1, \mathbf{X}_2) = f_{AA}(w_{AA}) + f_{cr}(w_0, w_{BA}, w_{AB}) + f_{BB}(w_{BB})$, where subscript cr stands for "crossed" states ((A, B) or (B, A)). See Fig. 2 for an illustration.

Stream AA is decoded first, regardless of whether the others can be decoded (this is done by treating all the other streams as interference). Then, both streams AB and BA including their common stream subscripted 0 can be decoded after removing the AA impact from their decoder inputs (treating the BB stream as interference). Finally, removing all above decoded streams allows decoding stream BB. From (2), we have

$$\begin{aligned} &R_{AA} \le 2\log\left(1 + \tfrac{\alpha_{AA} P}{\nu_a^{-1} + \bar{\alpha}_{AA} P}\right); \\ &R_{AA} + R_0 \le 2\log\left(1 + \tfrac{\alpha_{AA} P}{\nu_a^{-1} + \bar{\alpha}_{AA} P}\right) \\ &+ \log\left(1 + \tfrac{\alpha \alpha_{cr} P}{\nu_b^{-1} + (\bar{\alpha}\alpha_{cr} + \alpha_{BB})P}\right) + \log\left(1 + \tfrac{\alpha \alpha_{cr} P}{\nu_a^{-1} + (\bar{\alpha}\alpha_{cr} + \alpha_{BB})P}\right); \\ &R_{AA} + R_0 + R_{BA} = R_{AA} + R_0 + R_{AB} \\ &\le 2\log\left(1 + \tfrac{\alpha_{AA} P}{\nu_a^{-1} + \bar{\alpha}_{AA} P}\right) + \log\left(1 + \tfrac{\alpha \alpha_{cr} P}{\nu_a^{-1} + (\bar{\alpha}\alpha_{cr} + \alpha_{BB})P}\right) \\ &+ \log\left(1 + \tfrac{\alpha_{cr} P}{\nu_b^{-1} + \alpha_{BB} P}\right); \\ &R_{AA} + R_0 + R_{BA} + R_{AB} \\ &\le 2\log\left(1 + \tfrac{\alpha_{AA} P}{\nu_a^{-1} + \bar{\alpha}_{AA} P}\right) + \log\left(1 + \tfrac{\alpha_{cr} P}{\nu_b^{-1} + \alpha_{BB} P}\right) \\ &+ \log\left(1 + \tfrac{\alpha \alpha_{cr} P}{\nu_a^{-1} + (\bar{\alpha}\alpha_{cr} + \alpha_{BB})P}\right) + \log\left(1 + \tfrac{\bar{\alpha}\alpha_{cr} P}{\nu_b^{-1} + \alpha_{BB} P}\right); \\ &R_{AA} + R_0 + R_{BA} + R_{AB} + R_{BB} \\ &\le 2\log\left(1 + \tfrac{\alpha_{AA} P}{\nu_a^{-1} + \bar{\alpha}_{AA} P}\right) + \log\left(1 + \tfrac{\alpha_{cr} P}{\nu_b^{-1} + \alpha_{BB} P}\right) \\ &+ \log\left(1 + \tfrac{\alpha \alpha_{cr} P}{\nu_a^{-1} + (\bar{\alpha}\alpha_{cr} + \alpha_{BB})P}\right) + \log\left(1 + \tfrac{\bar{\alpha}\alpha_{cr} P}{\nu_b^{-1} + \alpha_{BB} P}\right) \\ &+ 2\log\left(1 + \tfrac{\alpha_{BB} P}{\nu_b^{-1}}\right); \end{aligned} \tag{3}$$

where $\alpha_{AA}, \alpha_{cr}, \alpha_{BB} \in [0, 1]$ are the relative power allocations for the subscripted letters $\alpha_{AA} + \alpha_{cr} + \alpha_{BB} = 1$, and $\alpha \in [0, 1]$ is the single user private power allocation within the unmatched channel.

B. Suggested Encoding and Decoding Scheme

Wrapping the extended model of Section IV-A with a message splitter at the transmitter and channel state dependent message multiplexer at the receiver enriches the domain. Fig. 3 illustrates the encoding and decoding schemes in full.

During decoding, the 4 possible channel states $\mathbf{S} = (S_1, S_2)$ impose different decoding capabilities. If $\mathbf{S} = (A, A)$, then $g_{AA}(\cdot)$ can reconstruct w_{AA} to achieve a total rate of R_{AA}. For $\mathbf{S} = (B, A)$, $g_{BA}(\cdot)$ is capable of reconstructing three messages (w_{AA}, w_0, w_{BA}) with sum rate of $R_{AA} + R_0 + R_{BA}$. Similarly for $\mathbf{S} = (A, B)$, $g_{AB}(\cdot)$ reconstructs (w_{AA}, w_0, w_{AB}) with sum rate $R_{AA} + R_0 + R_{AB}$. When both channels are permissive $\mathbf{S} = (B, B)$, all 5 messages $(w_{AA}, w_0, w_{BA}, w_{AB}, w_{BB})$ are reconstructed at $g_{BB}(\cdot)$ under the rate $R_{AA} + R_0 + R_{BA} + R_{AB} + R_{BB}$.

C. Average Sum Rate

Stitching up all cases with their probabilities, gives rise to the average rate of the parallel channel of

$$\begin{aligned} R_{\text{avg}} = P_A^2 R_{AA} &+ P_A P_B (R_{AA} + R_0 + R_{AB}) \\ &+ P_B P_A (R_{AA} + R_0 + R_{BA}) \\ &+ P_B^2 (R_{AA} + R_0 + R_{BA} + R_{AB} + R_{BB}). \end{aligned} \tag{4}$$

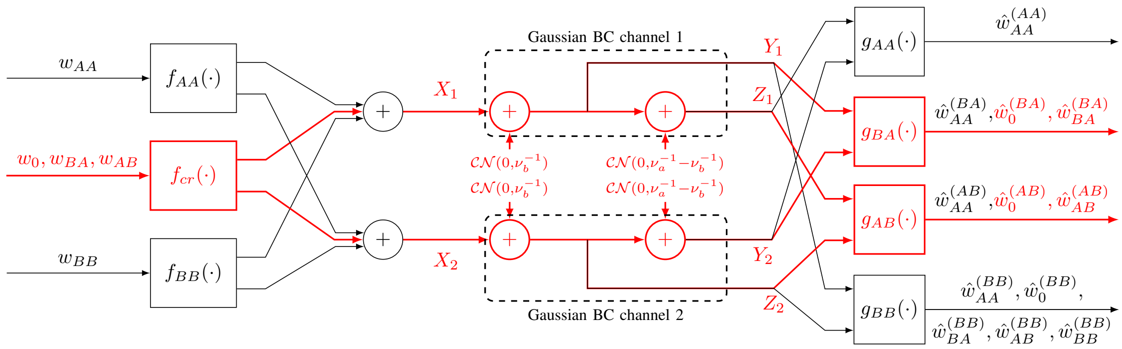

Fig. 2: Encoding-decoding scheme of the 2 receiver Gaussian degraded product broadcast channel with users: AA,AB,BA,BB

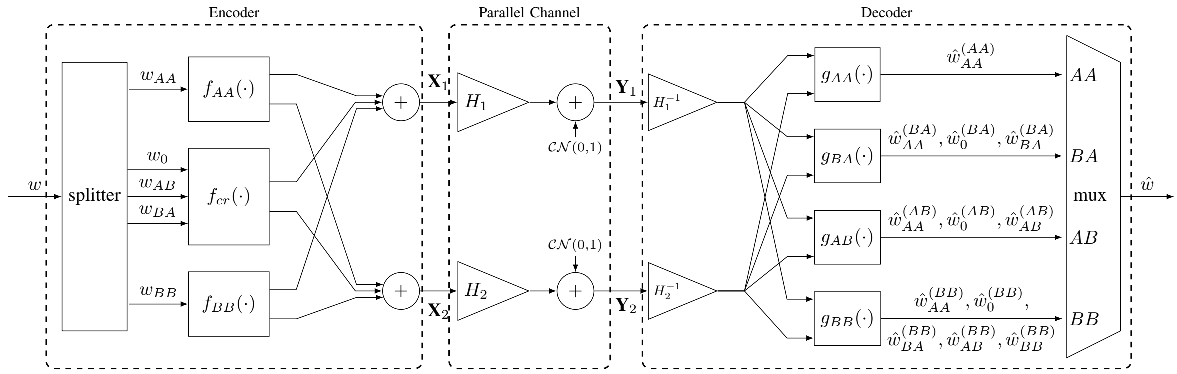

Fig. 3: Encoding and decoding scheme of the two receiver Gaussian degraded product broadcast channel broadcast approach

Using (3), and since both channels have identical statistics lead to $R_{AB} = R_{BA}$, and the achievable average rate is

$$\begin{aligned} R_{\text{avg}} &= 2(P_A+P_B)^2 \log(1+\nu_a P) + R_0(1-\alpha_{AA}) \\ &\quad + R_1(1-\alpha_{AA}-\alpha\alpha_{cr}) + R_2(1-\alpha_{AA}-\alpha_{cr}), \end{aligned} \tag{5}$$

where the new notations are

$$\begin{aligned} R_0(\alpha_0) &= [(P_A+P_B)^2 - P_A^2] \log(1+\nu_b\alpha_0 P) \\ &\quad - [(P_A+P_B)^2 + P_A^2] \log(1+\nu_a\alpha_0 P), \end{aligned} \tag{6}$$

$$\begin{aligned} R_1(\alpha_1) &= P_B^2 \log(1+\nu_b\alpha_1 P) \\ &\quad - [(P_A+P_B)^2 - P_A^2] \log(1+\nu_a\alpha_1 P), \end{aligned} \tag{7}$$

$$R_2(\alpha_2) = -2P_A P_B \log(1+\nu_b\alpha_2 P). \tag{8}$$

and the arguements $\alpha_0 = 1-\alpha_{AA}$, $\alpha_1 = 1-\alpha_{AA}-\alpha\alpha_{cr}$ and $\alpha_2 = 1-\alpha_{AA}-\alpha_{cr} = \alpha_{BB}$. Note that $R_0(\alpha_0)$ and $R_1(\alpha_1)$ are not obliged to be positive, as they can be negative for some scenarios, and $R_2(\alpha_2)$ is non-positive by definition.

Denoting the domain D' of valid power allocations vector $\boldsymbol{\alpha}' = [\alpha, \alpha_{AA}, \alpha_{cr}, \alpha_{BB}]^T \in [0,1]^4$ and the operator $[x]_+ = \max\{0, x\}$ yield the following.

Proposition 1. *The maximal sum rate of the symmetric two parallel two state channel over all power allocations is*

$$\begin{aligned} \max_{\boldsymbol{\alpha}' \in D'} R_{\text{avg}}(\boldsymbol{\alpha}') &= 2(P_A+P_B)^2 \log(1+\nu_a P) \\ &\quad + \max_{0\le\alpha_{AA}\le 1} \{R_0(1-\alpha_{AA}) + R_1(\alpha_1^{\text{opt}}(\alpha_{AA}))\}, \end{aligned}$$

where

$$\alpha_1^{\text{opt}}(\alpha_{AA}) = \max\{0, \min\{1-\alpha_{AA}, \alpha_1^*\}\}, \tag{9}$$

$$\alpha_1^* = \frac{P_B^2\nu_b - [(P_A+P_B)^2 - P_A^2]\nu_a}{[(P_A+P_B)^2 - P_A^2 - P_B^2]\nu_a\nu_b P}, \tag{10}$$

where the latter solves $\frac{\partial}{\partial\alpha_1} R_1(\alpha_1^*) = 0$.

Proof. Consider the transform $t' : D' \to D$ defined by $[\alpha_0, \alpha_1, \alpha_2]^T = \boldsymbol{\alpha} = t'(\boldsymbol{\alpha}'^T) = t'\left([\alpha, \alpha_{AA}, \alpha_{cr}, \alpha_{BB}]^T\right) = [1-\alpha_{AA}, 1-\alpha_{AA}-\alpha\alpha_{cr}, \alpha_{BB}]^T$, which is bijective, with inverse transform $t : D \to D'$ defined by $[\alpha, \alpha_{AA}, \alpha_{cr}, \alpha_{BB}]^T = \boldsymbol{\alpha}' = t'(\boldsymbol{\alpha}'^T) = t'\left([\alpha_0, \alpha_1, \alpha_2]^T\right) = [\frac{\alpha_0-\alpha_1}{\alpha_0-\alpha_2}, 1-\alpha_0, \alpha_0-\alpha_2, \alpha_2]^T$. Bijectiveness leads to

$$\begin{aligned} \max_{\boldsymbol{\alpha}' \in D'} R_{\text{avg}}(\boldsymbol{\alpha}') &= \max_{\boldsymbol{\alpha} \in D} \left\{ 2(P_A+P_B)^2 \log(1+\nu_a P) + \sum_{i=0}^{2} R_i(\alpha_i) \right\} \\ &= 2(P_A+P_B)^2 \log(1+\nu_a P) + \max_{\substack{\alpha_0,\alpha_1: \\ 0\le\alpha_1\le\alpha_0\le 1}} \{R_0(\alpha_0) + R_1(\alpha_1)\} \end{aligned}$$

The maximization of $R_2(\alpha_2)$ yields $\alpha_2^{\text{opt}} = 0$, as $R_2(\alpha_2)$ is a decreasing function. Further simplification gives,

$$\begin{aligned} \max_{\boldsymbol{\alpha}' \in D'} R_{\text{avg}}(\boldsymbol{\alpha}') &= 2(P_A+P_B)^2 \log(1+\nu_a P) \\ &\quad + \max_{0\le\alpha_{AA}\le 1} \left\{ R_0(1-\alpha_{AA}) + \max_{\substack{\alpha_1: \\ 0\le\alpha_1\le 1-\alpha_{AA}}} R_1(\alpha_1) \right\}. \end{aligned}$$

The inner maximization is done over α_1 while α_{AA} is fixed prior to the maximization. By taking the first derivative w.r.t α_1 and some calculus, optimality is achieved for (9). ■

Corollary 2. *The optimal power allocation for the state* (B, B) *is* $\alpha_{BB}^{\text{opt}} = 0$.

This is true for any set of parameters ν_a, ν_b, P_A, P_B, even if $P_B \to 1$ and $\nu_b \gg \nu_a$. Inherently, a penalty occurs when trying to exploit the double permissive state.

Corollary 3. *Under the optimal power allocation,* $\alpha^{\text{opt}}(\alpha_{AA}) = 1 - \alpha_1^{\text{opt}}(\alpha_{AA})/(1-\alpha_{AA})$.

This removes a degree of freedom in the optimization problem. Using these corollaries, and the notation $\boldsymbol{\alpha}' = [\alpha, \alpha_{AA}, \alpha_{cr}, \alpha_{BB}]^T$ instead of $\boldsymbol{\alpha} = [\alpha_0, \alpha_1, \alpha_2]^T$, we have:

Theorem 4. *The maximal sum rate of the symmetric two-parallel two-state channel over all allocations* $\boldsymbol{\alpha}' \in D'$ *is*

$$R_{\text{avg}}^{\text{opt}} = 2(P_A + P_B)^2 \log(1 + \nu_a P) + \max_{0 \le \alpha_{AA} \le 1} \left\{ R_0(1-\alpha_{AA}) + R_1((1-\alpha_{AA})\cdot(1-\alpha^{\text{opt}}(\alpha_{AA}))) \right\}$$

where

$$\alpha^{\text{opt}}(\alpha_{AA}) = \left[\min\left\{1, 1 - \frac{P_B^2 \nu_b - [(P_A+P_B)^2 - P_A^2]\nu_a}{2P_A \cdot P_B \cdot \nu_a \nu_b P(1-\alpha_{AA})}\right\}\right]_+. \quad (11)$$

Denoting the argument of the maximization as α_{AA}^{opt}*, the optimal power allocation vector is*

$$\boldsymbol{\alpha}'^{\text{opt}} = [\alpha^{\text{opt}}(\alpha_{AA}), \alpha_{AA}^{\text{opt}}, 1 - \alpha_{AA}^{\text{opt}}, 0]^T.$$

Proof. Use Prop. 1 and note that $\alpha_1 = 1 - \alpha_{AA} - \alpha\alpha_{cr} = (1 - \alpha_{AA})(1 - \alpha)$ for the optimal alocation $\alpha_{BB} = 0$. ■

D. Sub Optimal Schemes

For evaluation of the advantage of the joint α_{AA} and α, the following sub optimal schemes are introduced: a) independent broadcasting; b) privately broadcasting; and c) only common broadcasting.

Definition 5. *A scheme for which the encoder disjointly encodes different messages into each single channel of the parallel channel using the broadcast approach over the fading channel is denoted* ***independent broadcasting****.*

The broadcast approach for fading SISO channel (introduced in [8], elaborated in [2]) relies on two main operations: superposition coding by layering at the transmitter; and successive interference cancellation at the receiver. The maximal average sum rate of the symmetric two parallel two state channel under **independent broadcasting** is

$$R_{\text{avg}}^{\text{ind-bc,opt}} = 2(P_A + P_B)\log\left(\frac{1+\nu_a P}{1+\nu_a(1-\alpha^{\text{ind-bc,opt}})P}\right) + 2P_B \log\left(1 + \nu_b(1 - \alpha^{\text{ind-bc,opt}})P\right),$$

$$\alpha^{\text{bc,opt}} = \left[\min\left\{1, 1 - \frac{P_B\nu_b - (P_A+P_B)\nu_a}{P_A \nu_a \nu_b P}\right\}\right]_+. \quad (12)$$

Definition 6. *A scheme for which no power is allocated for the common stream in the* (B, A) *and* (A, B) *states (message* w_0*) is denoted* ***privately broadcasting****.*

This scheme is equivalent to setting $\alpha = 0$ in Theorem 4, thus allocating encoding power from the common stream ($R_0 = 0$) to the other streams R_{AA}, R_{AB}, R_{BA} and R_{BB} which achieves optimality for

$$\alpha_{AA}^{\text{prv-bc,opt}} = \left[\min\left\{1, 1 - \frac{[P_B - P_A]\nu_b - [P_B + P_A]\nu_a}{2P_A \nu_a \nu_b P}\right\}\right]_+.$$

Definition 7. *A scheme for which all of the crossed state power is allocated for common stream only (message* w_0*) and no power is allocated privately (no allocation for messages* w_{AB} *and* w_{BA}*) is denoted* ***only common broadcasting****.*

This scheme is equivalent to setting $\alpha = 1$ in Theorem 4, thus allocating encoding power from the private streams ($R_{AB} = R_{BA} = 0$) to the other streams R_{AA}, R_0 and R_{BB} which achieves optimality for

$$\alpha_{AA}^{\text{cmn-bc,opt}} = \left[\min\left\{1, 1 - \frac{[(P_A+P_B)^2 - P_A^2]\nu_b - [(P_A+P_B)^2 + P_A^2]\nu_a}{2P_A^2 \nu_a \nu_b P}\right\}\right]_+.$$

E. Numerical Results

Fig. 4 demonstrates the optimality of the proposed scheme (Theorem 4). The selected metric is the part of each scheme as a fraction of ergodic capacity. It is always superior in comparison to the other sub-optimal schemes, and captures a large portion of the ergodic capacity which stands as the upper bound. The sub-optimal methods inferior or superior to other sub-optimal methods, dependent on the parameters set. Some parameters sets can make them coincide for all SNR values. The gap to ergodic capacity does not change much, indicating that most coding gain is achieved via one of the classical broadcasting, and the specific one is parameters-set dependent.

V. SISO Block Fading

A. SISO consecutive block encoding model

Consider a block fading channel, as depicted at Fig. 5. Each n discrete time samples, a message w is to be encoded into the sequence $\mathbf{X} \in \mathbb{C}^n$, which enters the single input single output block fading channel satisfying the power constraint $\mathbb{E}|X|^2 \le P$ where X is the single letter random variable representation of the vector $\mathbf{X}$ and P is the power constraint $\mathbf{Y} = H\mathbf{X} + \mathbf{N}$. The channel gain $H \in \mathbb{C}$ is fixed within the n length block, and changes in-between blocks according to a priori known statistics P_H in a memoryless fashion. A complex normal noise is added, i.i.d. per channel output sample. The decoder is fully aware of the block gain (by channel sounding using pilot symbols) and reconstructs the message $\hat{w}$. The encoder has no way to know the channel realizations, yet has knowledge regarding its statistics P_H.

This setting, when allowing consecutive blocks *variable-to-fixed coding* [3] joint encoding, is actually a variant of the parallel MIMO single user case, where the diversity is over time blocks. Any development done so far can be applied on this special case. By allowing coding over two blocks at a time, the parallel channel model described till this section holds for this channel as well. The drawback is additional latency, yet only in the length of a single block, which in some use cases can be justified for the boos of achievable average rate.

B. A comment on Whiting [20]

The result in Theorem 4 differs from the one presented in [20] for the two-parallel two state channel. In [20] it is chosen to transmit only common information to the pairs (A, B) and (B, A). [20, eq. (39)] clearly states that for the crossed states (A, B) and (B, A) only common rate is used without justification. It is further claimed that this is an expected rate upper bound for some power allocation. Our result fully coincides with [20, eq. (39)] for $\alpha = 1$ rather than as in (9). However, this work proves that $\alpha = 1$ is suboptimal, and does not yield the maximal average rate. Furthermore, [20] does not notice that $\alpha_{BB} = 0$, whereas in this paper it is shown analytically to be optimal in Corollary 2.

VI. Conclusion

The broadcast approach for the parallel MIMO two state block fading channel is studied. The optimal scheme based on the concept of El-Gamal's degraded broadcast product channel, requires transmission of both private and common streams on two states (A, B) or (B, A). The expected rate is maximized

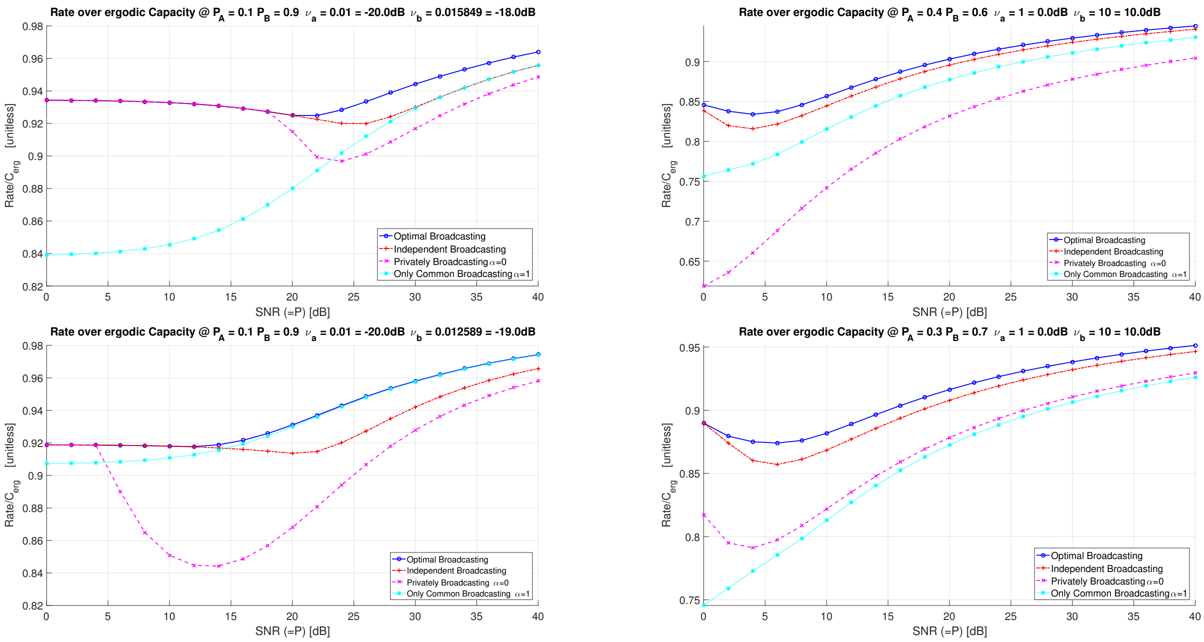

Fig. 4: Average sum rate as portion of the ergodic capacity of different schemes for several parameters-sets.

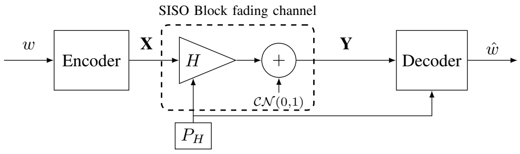

Fig. 5: The SISO n-length block fading channel and system.

analytically for layered transmission over the parallel channel. We demonstrate that the simple broadcast approach operating on each of the parallel channels separately achieves a significant portion of the optimal average rate. While the simple two-state parallel channel is considered here, the results apply directly to reduced latency constraints, that permit decoding over two fading blocks of a single two state fading channel. Evidently, extensions to a richer state spaces are called for, which may motivate new broadcast approach concepts of direct interest to future latency limited wireless systems. The framework considered motivates extensions where also the number of parallel channels received is random (adding thus a zero state), and this model may give rise to examine also secrecy constraints [22].

VII. Acknowledgment

This work has been supported by the European Union's Horizon 2020 Research And Innovation Programme, grant agreement no. 694630.

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