Bounds on the Capacity of the Multiple Access Diamond Channel with Cooperating Base-Stations

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***Abstract*—A diamond network is considered in which the central processor is connected, via backhaul noiseless links, to multiple conferencing base stations that communicate with a single user over a multiple access channel. We propose coding techniques along with lower and upper bounds on the capacity. Our achievability scheme uses a common cloud coding strategy based on the technique proposed by Wand, Wigger, and Zaidi (2018) and extends it beyond two relays. Our upper bounds generalize the method proposed by Bidokhti and Kramer (2016) and lead to new bounds for the multiple conferencing relay setting. Specializing our upper bounds to the two relay scenario, we provide new bounds and improve the state-of-the-art.**

I. Introduction

As the demand for steadily growing data transmission increases, capacity stands as the main challenge for every emerging generation of mobile networks [1]. In this regard, ultra-dense cell deployment with cooperative operations is expected to become a key technology for enabling modern, reliable, ultra-bandwidth, scalable, and fast communication systems [2]. Initially proposed in [3], C-RAN is an emerging network architecture that enables large-scale cooperation among base stations [4]. As opposed to traditional architectures where radio and baseband processing functionality is solely implemented inside a base station (BS), in C-RAN, the BS performs digital processing, digital to analog conversion, analog to digital conversion, power amplification, and filtering, while baseband processing is performed in a central processor (CP) connected to multiple BSs via finite capacity backhaul links. The characteristic of this architecture makes C-RAN capable of dealing with intensive inter-cell interference in future ultra-dense, multi-tier networks [5]. Comprehensive surveys on C-RAN can be found in [6]–[8]. This work will consider a multi-hop architecture of C-RAN's downlink with multiple conferencing BSs. Such architecture, which gives an excellent value-for-money, has the potential to provide a very-high capacity gains.

In the past two decades, various representing models have been considered in the information theory literature for the uplink of C-RANs. The problem of point-to-point communication over the broadcast channel with the help of two BSs connected via finite capacity links to the receiver has been addressed in [9]. The multi-user multi-BS model for the uplink C-RAN communication with oblivious BSs was considered in [10]. In that model, the BS nodes are constrained to operate without knowing the users' codebook. Optimal coding schemes were presented, the capacity was determined under the oblivious processing regime, and connection to the information bottleneck method was settled. Also, a complete characterization of the capacity region was provided for the Gaussian signaling setting for a more general setting. In our work we consider a dual problem of multi-BSs setting in the downlink of C-RAN.

The diamond channel plays a central role as a simple yet insightful model to the downlink of C-RAN. The Gaussian multiple access diamond channel was studied in [11]. The downlink of symmetric C-RANs with multiple, non-collaborating BSs and a single receiver was studied in [12]. A generalized compression strategy for the downlink of C-RANs was proposed in [13]. An improved outer bound on the capacity of the downlink C-RAN, based on the generalized Entropy Power Inequality (EPI), has been obtained in [14]. While in the uplink C-RAN, obliviousness of the BSs is the main driving force for considering such a setting, defining the oblivious BS processing region to the downlink C-RAN is more challenging since information is conveyed first to the BSs. Thus, introducing oblivious BSs to the downlink of C-RAN remains an open problem.

In classical cellular communication systems, the uplink is modeled via a multiple-access channel, and the downlink is modeled using a broadcast channel. The capacity region of the Gaussian vector broadcast channel subject to a sum power constraint is precisely the same as the capacity region of a reciprocal multiple-access channel [15], [16]. Therefore, it is compelling to find such an elegant relation, termed "uplink-downlink duality," in C-RAN models. Capacity approximation within a constant gap of the fronthaul-limited uplink and downlink C-RAN using noisy network coding and distributed decode-forward has been recently shown [17]. An elegant uplink-downlink duality property for the Gaussian C-RAN has been identified in [18]. Specifically, it was established that under the same total transmit power constraint and individual fronthaul rate constraints, the achievable rate regions of the Gaussian multiple-access and broadcast relay channels are identical. An uplink-downlink duality for C-RAN with cooperating BSs has not yet been found.

A communication network consisting of k-transmitters over a multiple access channel (MAC) with encoder-level cooperation and single receiver node has been considered in [19]. It was shown that even a little bit of cooperation could increase rates significantly [20]. For example, cooperation can be used to design correlative signals by exploiting half of the link capacity, simply by sending two separate copies of the common message with the direct links and using the cooperation link

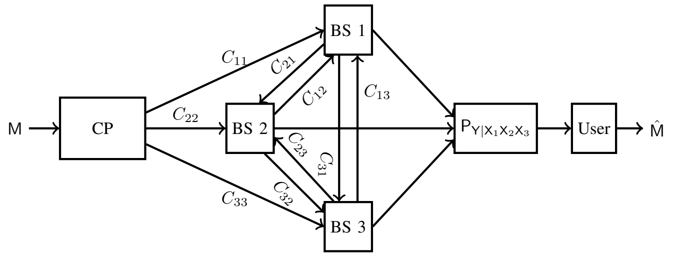

Fig. 1: Downlink C-RAN with BS cooperation: 3 base stations and one mobile user.

to combine them at the BSs. Therefore, incorporating inter-relay cooperation has the potential to increase information rates further at a low cost. A class of diamond networks with conferencing BSs, C-RAN model with BSs cooperation, was proposed in [21]. The inclusion of cooperation between the BSs in the downlink C-RAN model with a pair of mobile users has been addressed in [22], where various coding schemes are proposed and compared. In particular, [22] generalizes the schemes of [11], [23], and [21] to the case of cooperating BSs.

In this work, we study point-to-point communication with multiple cooperating BSs. Note that this is not a conferencing MAC model [24], where the BSs can cooperate and each BS has an independent message to transmit, but instead has a nontrivial encoding complexity structure, and different coding techniques are employed. Moreover, the BSs are partially co-operative in C-RANs because of the central processor and the back-haul connections, with conferencing being an additional form of collaboration that can be implemented locally by deploying direct communication links between the BSs. Our main contribution is the extension of the two conferencing BSs setting [21], [22], and the inclusion of cooperation in the multiple BS setup studied in [12]. We derive lower and upper bounds and evaluate them for the Additive White Gaussian Noise (AWGN) channel. Furthermore, we show numerically that our upper bound outperform the state-of-the-art for the two BSs case. Omitted proofs can be found in the full version of this paper [25].

II. Problem Formulation

Consider the downlink 3-BS 1-user C-RAN with BS cooperation depicted in Fig. 1. The network consists of one CP, three BSs, and one Mobile User (MU). The CP communicates with the three BSs through individual noiseless fronthaul links of finite capacities. Denote by C_{jj} the link's capacity from the CP to BS j, for $j \in [3]$. In addition, the three BSs can also communicate with each other through individual noiseless fronthaul links of finite capacities. Let $\mathcal{I}_j \triangleq \{1,2,3\} \setminus j$. Denote by C_{ij} the link's capacity from BS j to BS $i \in \mathcal{I}_j$. The network from the BSs to the MU is modeled as a Discrete Memoryless Multiple Access Channel (DM-MAC) $\langle \mathcal{X}_1 \times \mathcal{X}_2 \times \mathcal{X}_3, \mathsf{P}_{\mathsf{Y}|\mathsf{X}_1\mathsf{X}_2\mathsf{X}_3}, \mathcal{Y}\rangle$ that consists of four finite sets $\mathcal{X}_1, \mathcal{X}_2, \mathcal{X}_3, \mathcal{Y}$, and a collection of the conditional probability mass functions (pmf) $\mathsf{P}_{\mathsf{Y}|\mathsf{X}_1\mathsf{X}_2\mathsf{X}_3}$.

With the help of the three BSs, the CP wishes to communicate a message M to the MU. Assume that M is uniformly distributed over $\mathcal{I}_R^{(n)}$, where $\mathcal{I}_R^{(n)}$ is the set of consecutive integers from 1 to 2^{nR}. This paper restricts attention to information processing on a block-by-block basis. Each block consists of a sequence of n symbols. The entire communication is divided into three sequential phases:

1) *CP to BSs:* The CP conveys three sequences $(\mathsf{S}_1^n, \mathsf{S}_2^n, \mathsf{S}_3^n) = f_0(\mathsf{M})$ to BS 1, BS 2, and BS 3 respectively, where $f_0 : \mathcal{I}_R^{(n)} \to \mathcal{I}_{C_{11}}^{(n)} \times \mathcal{I}_{C_{22}}^{(n)} \times \mathcal{I}_{C_{33}}^{(n)}$ is the encoder of the CP.
2) *BS-to-BS conferencing cooperation:* After receiving S_j^n, $j \in [3]$, the BSs can communicate through K rounds in a round-robin fashion. At round k, BS j sends $\{\mathsf{V}_{ij}\}_{i\in\mathcal{I}_j}$ to the other BSs based on S_j^n and the messages from the other BSs in previous round, i.e, $\{\mathsf{V}_{ij,k}\}_{i\in\mathcal{I}_j} = f_j(\mathsf{S}_j^n, \{\mathsf{V}_{ji}^{k-1}\}_{i\in\mathcal{I}_j})$.
3) *BSs to MU*: BS j transmits a sequence $\mathsf{X}_j^n = g_j(\mathsf{S}_j^n, \{\mathsf{V}_{ji}\}_{i\in\mathcal{I}_j})$ over the DM-MAC, where, g_j is the channel encoder of BS j, $j \in \{1,2,3\}$.

Upon receiving the sequence Y^n, the MU assigns an estimate $\hat{\mathsf{M}} = d(\mathsf{Y}^n)$ of the message M where $d : \mathcal{Y}^n \to \mathcal{I}_R^{(n)}$. The collection of the encoders $f_0, f_1, f_2, f_3, g_1, g_2, g_3$ and the decoder d constitute a $(2^{nR}, n)$ code.

The average probability of error is defined as $P_e^{(n)} = \mathsf{P}\left(\hat{\mathsf{M}} \neq \mathsf{M}\right)$. A rate R is said to be achievable if there exists a sequence of $(2^{nR}, n)$ codes such that $\lim_{n\to\infty} P_e^{(n)} = 0$. The capacity C is the supremum of all achievable rates.

Remark 1: We have assumed a three BSs setting since a more general number of BSs results in intractable expressions due to multiple possible paths of information transmission between the CP and the BSs.

The results we obtain here for the DM-MAC can be readily adapted for MAC with continuous input/output alphabets and input costs applying discretization arguments as in [26, Sec. 3.4.1]. More specifically, in this work, we consider the Symmetric Gaussian MAC, which is defined by the following input-output relation:

$$\mathsf{Y} = \mathsf{X}_1 + \mathsf{X}_2 + \mathsf{X}_3 + \mathsf{Z}, \tag{1}$$

where $\mathsf{X}_k \in \mathbb{R}$ is the channel input from BS k, Y is the channel output, and $\mathsf{Z} \sim \mathcal{N}(0,1)$ is additive noise. In addition, each BS has to satisfy an average power constraint P, i.e., $\frac{1}{n}\sum_{i=1}^n \mathbb{E}\,[\mathsf{X}]_{ki}^2 \leq P$, for all $k \in \{1,2,3\}$. Furthermore, the noiseless links have symmetric capacities, i.e., $C_{11} = C_{22} = C_{33} = C$, and $C_{12} = C_{21} = C_{13} = C_{31} = C_{23} = C_{32} = C_0$.

III. The case of 2 relays

The downlink 2-BS 1-user C-RAN, with BSs cooperation [21] can be considered as a special case of the downlink 3-BS 1-user C-RAN defined in Section II by setting $C_{33} = C_{13} = C_{31} = C_{23} = C_{32} = 0$.

A. Bounds on Capacity

The following lower bound has been obtained in [22].

Lemma 1 ([22, Corollary 5]): Any rate R is achievable for the downlink 2-BS 1-user C-RAN with BS cooperation if there exists some pmf $\mathsf{P}_{\mathsf{UX}_1\mathsf{X}_2\mathsf{Y}} = \mathsf{P}_{\mathsf{UX}_1\mathsf{X}_2}\mathsf{P}_{\mathsf{Y}|\mathsf{X}_1\mathsf{X}_2}$, $\mathsf{U} \in \mathcal{U}$ and $|\mathcal{U}| \leq \min\{|\mathcal{X}_1| \cdot |\mathcal{X}_2| + 2, |\mathcal{Y}| + 4\}$ such that

$$R < \min \begin{cases} C_{11} + C_{22} - I(\mathsf{X}_1; \mathsf{X}_2|\mathsf{U}), \\ C_{11} + C_{12} + I(\mathsf{X}_2; \mathsf{Y}|\mathsf{U}, \mathsf{X}_1), \\ C_{22} + C_{21} + I(\mathsf{X}_1; \mathsf{Y}|\mathsf{U}, \mathsf{X}_2), \\ I(\mathsf{X}_1, \mathsf{X}_2; \mathsf{Y}), \\ \frac{1}{2}[C_{11} + C_{22} + C_{12} + C_{21} + I(\mathsf{X}_1, \mathsf{X}_2; \mathsf{Y}|\mathsf{U}) - I(\mathsf{X}_1; \mathsf{X}_2|\mathsf{U})]. \end{cases}$$

This bound was initially proposed in [22] and it recovers the achievability results from [21, Theorem 2] by setting $C_{12} = C_{21} = C_0$. It can be further shown that the proposed scheme also retrieves the achievable rate for the downlink 2-BS 1-user C-RAN without BS cooperation considered in [11], [23].

The following computable upper bounds presented here are based on the ideas first developed in [23].

Theorem 1: An upper bound on the capacity of the diamond channel with two conferencing relays is given by $C < \max_{\mathsf{P}_{\mathsf{X}_1\mathsf{X}_2}} \min_{\mathsf{P}_{\mathsf{V}|\mathsf{UX}_1\mathsf{X}_2\mathsf{Y}}} \max_{\mathsf{P}_{\mathsf{U}|\mathsf{X}_1\mathsf{X}_2}} \beta_1$, where

$$\beta_1 \triangleq \min \begin{cases} C_{11} + C_{22}, \\ C_{11} + C_{12} + I(\mathsf{X}_2; \mathsf{Y}|\mathsf{U}, \mathsf{X}_1), \\ C_{22} + C_{21} + I(\mathsf{X}_1; \mathsf{Y}|\mathsf{U}, \mathsf{X}_2), \\ I(\mathsf{X}_1, \mathsf{X}_2; \mathsf{Y}), \\ C_{12} + C_{21} + I(\mathsf{X}_1, \mathsf{X}_2; \mathsf{Y}|\mathsf{U}), \\ \frac{1}{2}\begin{bmatrix} C_{11} + C_2 + C_{12} + C_{21} + I(\mathsf{X}_1; \mathsf{V}|\mathsf{U}, \mathsf{X}_2) \\ + I(\mathsf{X}_1, \mathsf{X}_2; \mathsf{Y}|\mathsf{U}, \mathsf{V}) + I(\mathsf{X}_2; \mathsf{V}|\mathsf{U}, \mathsf{X}_1) \end{bmatrix}. \end{cases} \tag{2}$$

Remark 2: Choosing $\mathsf{P}_{\mathsf{V}|\mathsf{U},\mathsf{X}_1,\mathsf{X}_2,\mathsf{Y}} = \mathsf{P}_{\mathsf{V}|\mathsf{Y}}$, the last term in the brackets of (2) can be equivalently rewritten as

$$C \leq \frac{1}{2}\begin{bmatrix} C_{11} + C_{22} + C_{12} + C_{21} + I(\mathsf{X}_1, \mathsf{X}_2; \mathsf{Y}|\mathsf{U}) \\ + I(\mathsf{X}_1; \mathsf{X}_2|\mathsf{U}, \mathsf{V}) - I(\mathsf{X}_1; \mathsf{X}_2|\mathsf{U}) \end{bmatrix}. \tag{3}$$

We will exploit this representation to design V that minimizes $I(\mathsf{X}_1; \mathsf{X}_2|\mathsf{U}, \mathsf{V})$.

Remark 3: The bound in Thm. 1 is based on [23, Thm. 2]. Note that it coincides with [21, Thm. 1] by setting $\mathcal{U} = \emptyset$, but, the last term in (3) suggests it may not be the optimal choice. Thus, this bound is a promising candidate.

Our next upper bound is based on [23, Thm. 3]. This bound also incorporates the collaborative nature of the problem and therefore is tighter then the respective cut-set bound.

Theorem 2: An upper bound on the capacity of 2-BSs 1-user network with conferencing relays is given by

$$C < \max_{\mathsf{P}_{\mathsf{X}_1\mathsf{X}_2}} \min_{\mathsf{P}_{\mathsf{V}|\mathsf{Y}}} \max_{\mathsf{P}_{\mathsf{UT}|\mathsf{X}_1\mathsf{X}_2}} \beta_2,$$

where

$$\beta_2 = \min \left\{ \begin{matrix} C_{11} + C_{22}, \\ C_{11} + C_{12} + I(\mathsf{X}_2; \mathsf{Y}|\mathsf{U}, \mathsf{X}_1, \mathsf{T}), \\ C_{22} + C_{21} + I(\mathsf{X}_1; \mathsf{Y}|\mathsf{U}, \mathsf{X}_2, \mathsf{T}), \\ I(\mathsf{X}_1, \mathsf{X}_2; \mathsf{Y}), \\ C_{12} + C_{21} + I(\mathsf{X}_1, \mathsf{X}_2; \mathsf{Y}|\mathsf{U}, \mathsf{T}), \\ C_{11} + C_{22} - I(\mathsf{X}_1, \mathsf{X}_2; \mathsf{V}|\mathsf{U}, \mathsf{T}) + \\ I(\mathsf{X}_1; \mathsf{V}|\mathsf{U}, \mathsf{X}_2, \mathsf{T}) + I(\mathsf{X}_2; \mathsf{V}|\mathsf{U}, \mathsf{X}_1, \mathsf{T}). \end{matrix} \right\} \tag{4}$$

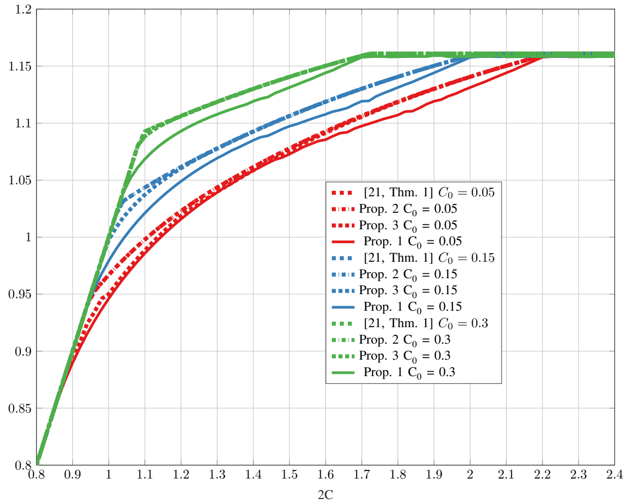

Fig. 2: Lower and upper bounds for 2 relays scenario from Props. 1, 2 and 3, with various values of C_0 and $P = 1$ vs C.

B. Example: Gaussian MAC

In this section we provide upper and lower bounds on the capacity of the 2-BS 1-user C-RAN with a symmetric Gaussian MAC, defined by (1) when $\mathsf{X}_3 = 0$. Plugging jointly Gaussian $(\mathsf{U}, \mathsf{X}_1, \mathsf{X}_2)$ in Lemma 1, we obtain the following achievable rate.

Proposition 1: For the Symmetric Gaussian diamond MAC with two conferencing relays, rate R is achievable if for some $0 \leq \rho \leq 1$, $0 \leq \rho_c \leq \sqrt{\frac{1+\rho}{2}}$, it satisfies

$$R \leq \min \begin{cases} 2C - \frac{1}{2}\log\left(\frac{(1-\rho_c^2)^2}{(1-\rho)(1+\rho-2\rho_c^2)}\right), \\ C + C_0 + \frac{1}{2}\log\left(\frac{1-\rho_c^2+(1-\rho)(1+\rho-2\rho_c^2)P}{1-\rho_c^2}\right), \\ \frac{1}{2}\log\left(1 + 2(1+\rho)P\right), \\ C + C_0 + \frac{1}{4}\log\left(\frac{[1+2(1+\rho-2\rho_c^2)P](1-\rho)(1+\rho-2\rho_c^2)}{(1-\rho_c^2)^2}\right). \end{cases}$$

We proceed to derive upper bounds for the symmetric case. We choose V to be a noisy version of Y, i.e., $\mathsf{U} = \mathsf{Y} + \mathsf{W}$ where $\mathsf{W} \sim (0, N)$. The following upper bound is a specialization of Thm. 1 to a Gaussian setting, utilizing Rem. 2 and choosing N such that $I(\mathsf{X}_1; \mathsf{X}_2|\mathsf{U}, \mathsf{V})$ is zero if possible.

Proposition 2: Rate R is achievable for the Symmetric Gaussian Diamond MAC only if it satisfies the following constraints for some $0 \leq \rho_c \leq \sqrt{\frac{1+\rho}{2}}$, $0 \leq \rho \leq \rho^*$:

$$R \leq \min \begin{cases} 2C, \\ C + C_0 + \frac{1}{2}\log\left(\frac{1-\rho_c^2+(1-\rho)(1+\rho-2\rho_c^2)P}{1-\rho_c^2}\right), \\ \frac{1}{2}\log\left(1 + 2(1+\rho)P\right), \\ 2C_0 + \frac{1}{2}\log\left(1 + 2(1+\rho-2\rho_c^2)P\right), \\ C + C_0 + \frac{1}{4}\log\left(\frac{[1+2(1+\rho-2\rho_c^2)P](1-\rho)(1+\rho-2\rho_c^2)}{(1-\rho_c^2)^2}\right), \end{cases} \tag{5}$$

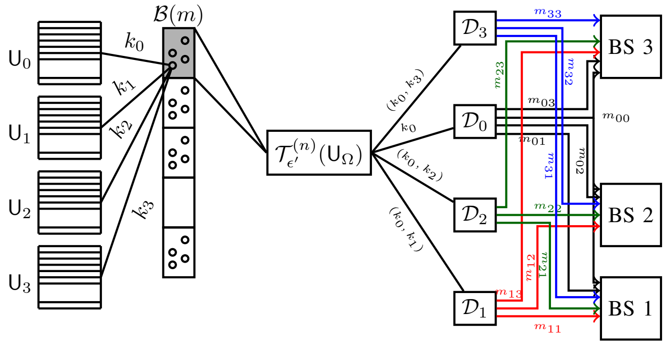

Fig. 3: Illustration of the encoding operation at the central processor in the SCC scheme.

and for $\rho > \rho^*$ we have

$$R \leq \min \begin{cases} 2C, \\ C + C_0 + \frac{1}{2}\log\left(\frac{1-\rho_c^2+(1-\rho)(1+\rho-2\rho_c^2)P}{1-\rho_c^2}\right), \\ \frac{1}{2}\log\left(1+2(1+\rho)P\right), \\ 2C_0 + \frac{1}{2}\log\left(1+2(1+\rho-2\rho_c^2)P\right), \end{cases} \tag{6}$$

where $\rho^* \triangleq \rho_c^2 + \sqrt{1 + \frac{1}{4P^2} - 2\rho_c^2 + \rho_c^4} - \frac{1}{2P}$.

Remark 4: We might improve over the upper bound found in [21] if $\rho_c^* > 0$. Note that if $\rho > \rho^*$, then the maximum of every term in (6) is achieved with $\rho_c^* = 0$. Thus, the interesting regime is when $\rho < \rho^*$ and the optimal $\rho_c^* > 0$. The only term that may contribute to this outcome is the last term in (5). The problem is that this term is coupled with

$$C + C_0 + \frac{1}{2}\log\left(\frac{1-\rho_c^2+(1-\rho)(1+\rho-2\rho_c^2)P}{1-\rho_c^2}\right),$$

which is maximized with $\rho_c^* = 0$ for any fixed ρ. Thus, it is very challenging to find the specific channel parameters for which the optimal $\rho_c > 0$.

Finally, we specialize Thm. 2 for the Gaussian setting.

Proposition 3: Rate R is achievable for the Symmetric Gaussian Diamond MAC only if it satisfies the following constraints for some $0 \leq \rho_c \leq \sqrt{\frac{1+\rho}{2}}$, $0 \leq \rho \leq 1$, $R \leq \min_{N\geq 0} \min \beta_3$, where

$$\beta_3 = \begin{cases} 2C, \\ C + C_0 + \frac{1}{2}\log\left(\frac{1-\rho_c^2+(1-\rho)(1+\rho-2\rho_c^2)P}{1-\rho_c^2}\right), \\ \frac{1}{2}\log\left(1+2(1+\rho)P\right), \\ 2C_0 + \frac{1}{2}\log\left(1+2(1+\rho-2\rho_c^2)P\right), \\ \frac{1}{2}\log\left[\sqrt{\frac{2^{4(C+C_0)}[\bar{\rho}_c^2(1+N)+\bar{\rho}(1+\rho-2\rho_c^2)P]^2}{(1-\rho_c^2)^2(1+N)} + \frac{2^{8C_0}N^2}{4}} - \frac{2^{4C_0}N}{2}\right]. \end{cases}$$

We further give a representative evaluation of the proposed bounds from Props. 1, 2 and 3 for $P = 1$ in Fig. 2. One observes that there is no significant contribution of Prop. 2 over [21, Thm . 1], but the bound from Prop. 3 gives the state-of-the-art performance over the regime of simulated channel parameters.

IV. The Case of 3 Relays - Symmetric

A. Lower Bounds

First, let us give a high-level overview of the proposed coding strategy. Consider the set $\Omega = \{0, 1, 2, 3\}$, fix a joint pmf P_{U_Ω} and independently generate 4 codebooks with sizes $\mathcal{I}_{R_\omega}^{(n)}$ from the marginals P_{U_ω}, $\forall \omega \in \Omega$. Each message $m \in \mathcal{I}_R^{(n)}$ is associated with a unique bin $\mathcal{B}(m)$ of index tuples $k_\Omega \triangleq (k_0, k_1, k_2, k_3)$, which are indices of the corresponding dictionaries. Then, given m, we apply joint typicality encoding to find index tuples $k_\Omega \in \mathcal{B}(m)$, such that $(U_\Omega^n(k_\Omega))$ are jointly typical. Subsequently, those index tuples are then sent to the respective BSs. Here, the fact that cooperation exists between the BSs is used to increase the effective rate from the CP to BSs, since there are multiple paths to transmit information to each one of the BSs. Finally, the MU applies joint typicality decoding to recover k_Ω and then the message m. The encoding architecture is illustrated in Fig. 3. The resulting achievability rate is presented in the following theorem.

Theorem 3: Let $\Omega = \{0, 1, 2, 3\}$. A rate R is achievable for the downlink 3-BS 1-user C-RAN with BS cooperation if there exist some rates $R_\omega \geq 0$, $\omega \in \Omega$, some joint pmf P_{U_Ω} and some functions $x_1(u_0, u_1)$, $x_2(u_0, u_2)$, $x_3(u_0, u_3)$, such that for all $\mathcal{S} \subseteq \Omega$ satisfying $|\mathcal{S}| \geq 1$, the following rate constraints hold:

- $\mathbb{1}\{\mathcal{S} = \Omega\}R < \sum_{\omega\in\mathcal{S}} R_\omega - \Gamma(U_\mathcal{S})$;
- $\sum_{\omega\in\mathcal{S}} R_\omega < I(U_\mathcal{S}; U_{\mathcal{S}^c}, Y) + \Gamma(U_\mathcal{S})$;
- $\sum_{l\in\Omega} C_{kl} \geq R_0 + R_k - I(U_0; U_k) \quad \forall k \in \Omega$;
- $\sum_{k\in\Omega} C_{kk} \geq R_0 + \sum_{k\in\Omega} R_k - I(U_0; U_k)$.

where $\Gamma(U_\mathcal{S}) \triangleq \sum_{\omega\in\mathcal{S}} H(U_\omega) - H(U_\mathcal{S})$.

Evaluation of the above rate for a specific channel is very difficult. Thus, we consider the following corollary where we restrict the correlation structure to be $U_0 = U$ and $U_k = X_k$ for $k \in [3]$.

Corollary 1: Rate R is achievable for the downlink 3-BS 1-user C-RAN with cooperation if there exists $P_{UX_1X_2X_3}$ such that the following holds for any $\mathcal{S} \subset [3]$:

$$R \leq \min \begin{cases} \sum_{k\in\mathcal{S},\omega\in[3]} C_{k\omega} + I(X_{\mathcal{S}^c}; Y|U, X_\mathcal{S}) - \Gamma(X_\mathcal{S}|U), |\mathcal{S}| \geq 1, \\ I(X_{[3]}; Y), \quad \sum_{\omega\in[3]} C_{\omega\omega} - \Gamma(X_\Omega|U) \\ \frac{1}{2}\left[\begin{array}{l} \sum_{k\in\mathcal{S},\omega\in[3]} C_{k\omega} + I(X_{\mathcal{S}^c}; Y|U, X_\mathcal{S}) \\ \qquad + I(X_{[3]}; Y|U) - \Gamma(X_\mathcal{S}|U) \end{array}\right], |\mathcal{S}| = 2 \\ \frac{1}{3}\left[\sum_{\omega\omega'\in[3]} C_{\omega\omega'} + 2I(X_{[3]}; Y|U) - \Gamma(X_{[3]}|U)\right]. \end{cases} \tag{7}$$

Remark 5: Our coding scheme for the three relays setting has an implicit symmetry assumption, namely, fixing the rates corresponding to the third node to zero results in congestion of the common rate. Thus, Cor. 1 cannot be directly applied to the two relays scenario. Nevertheless, the lines of the equations in the lower bound for two relays of Lemma 1 are comparable to the lines of the lower bound for three relays Cor. 1, except the last line of (7), reflecting the similarities among the coding schemes.

B. Upper Bound

We present here a new upper bound that extends the bounds found in [21], [23] for the three BSs scenario with cooperation.

Theorem 4: An upper bound on the capacity of 3-BSs 1-MU network with conferencing relays is given by

$$R \leq \max_{\mathsf{P}_{\mathsf{X}_1\mathsf{X}_2}} \min_{\mathsf{P}_{\mathsf{V}|\mathsf{Y}}} \max_{\mathsf{P}_{\mathsf{UT}|\mathsf{X}_1\mathsf{X}_2}} \beta_4,$$

where for every $\mathcal{S} \subset [3]$

$$\beta_4 = \min \begin{cases} \sum_{\omega\in\mathcal{S}} C_{\omega\omega} + \sum_{\substack{\omega\in[3]\\ \omega'\neq\omega\in\mathcal{S}^c}} C_{\omega\omega'} + I(\mathsf{X}_{\mathcal{S}^c};\mathsf{Y}|\mathsf{U},\mathsf{X}_{\mathcal{S}},\mathsf{T}), \\ I(\mathsf{X}_{[3]};\mathsf{Y}), \quad \sum_{\omega\in[3]} C_{\omega\omega}, \\ \sum_{\omega\in[3]} C_{\omega\omega} - 2I(\mathsf{X}_{[3]};\mathsf{V}|\mathsf{U},\mathsf{T}) + I(\mathsf{X}_1,\mathsf{X}_2;\mathsf{V}|\mathsf{U},\mathsf{T},\mathsf{X}_3) \\ + I(\mathsf{X}_1,\mathsf{X}_3;\mathsf{V}|\mathsf{U},\mathsf{T},\mathsf{X}_2) + I(\mathsf{X}_2,\mathsf{X}_3;\mathsf{V}|\mathsf{U},\mathsf{T},\mathsf{X}_1). \end{cases} \tag{8}$$

This result follows by combining methods used to single-letterize the expressions in the proof of [23, Thm. 3].

C. Gaussian MAC

In this section we evaluate Cor. 1 and Thm. 4 for the Symmetric Gaussian MAC. We define the following functions:

$$F_1 = \frac{1}{2}\log\frac{(1-\rho_c^2)^3}{(1-\rho)^2(1+2\rho-3\rho_c^2)}$$

$$F_2 = \frac{1}{2}\log\left(\frac{1-\rho_c^2+2\left(1-2\rho^2-3\rho_c^2+\rho+3\rho\rho_c^2\right)P}{1-\rho_c^2}\right)$$

$$F_3 = \frac{1}{2}\log\left(\frac{1+\rho-2\rho_c^2+\left(1+\rho-2\rho^2-3\rho_c^2-3\rho\rho_c^2\right)P}{1+\rho-2\rho_c^2}\right)$$

$$F_4 = \frac{1}{2}\log\frac{(1-\rho_c^2)^2}{(1+\rho)(1-\rho+2\rho_c^2)}, F_5 = \frac{1}{2}\log[1+3(1+2\rho)P]$$

$$F_6 = \frac{1}{2}\log\left(1+3(1+2\rho-3\rho_c^2)P\right)$$

$$F_7 = -\log\left(2^{2(R-6C_0)}+N\right) - \frac{1}{2}\log(1+N) + \frac{3}{2}\log\left(\frac{(1-\rho_c^2)(1+N)+2(1-\rho)(1+2\rho-3\rho_c^2)P}{1-\rho_c^2}\right).$$

Utilizing Cor. 1 for the symmetric Gaussian setting in (1), we obtain the following lower bound.

Proposition 4: The rate R is achievable if it satisfies the following constraints for some non-negative parameter ρ, $0 \leq \rho \leq 1$, $0 \leq \rho_c \leq \min\left\{\sqrt{\frac{1+\rho}{2}}, \sqrt{\frac{1+2\rho}{3}}\right\}$:

$$R < \min \begin{Bmatrix} 3C - F_1, C + 2C_0 + F_2, \\ 2C + 4C_0 + F_3 - F_4, \\ F_5, \frac{1}{2}\left[C + 2C_0 + F_2 + F_6\right], \\ C + 2C_0 + \frac{1}{2}\left[F_3 + F_6 - F_4\right], \\ C + 2C_0 + \frac{1}{3}\left[2F_6 - F_1\right]. \end{Bmatrix} \tag{9}$$

For the following upper bound, we choose V to be a noisy version of Y, i.e., $\mathsf{U} = \mathsf{Y} + \mathsf{W}$ where $\mathsf{W} \sim (0, N)$. We then specialize Thm. 4 for the symmetric Gaussian setting.

Proposition 5: Rate R is achievable only if it satisfies the following constraints for some $0 \leq \rho, \rho_c \leq 1$, such that $0 \leq \rho_c \leq \min\left\{\sqrt{\frac{1+\rho}{2}}, \sqrt{\frac{1+2\rho}{3}}\right\}$

$$R < \min \begin{Bmatrix} 3C, C + 2C_0 + F_2, \\ 2C + 4C_0 + F_3, F_5, 3C + F_7. \end{Bmatrix} \tag{10}$$

We provide an evaluation of the proposed bounds from Props. 4, and 5 for $P = 1$ in Fig. 4. One observes that our upper bound from Prop. 5 outperforms the cut-set bound for all region of simulated channel parameters. Furthermore, the gap between the upper and lower bounds decreases as the capacity of the cooperation link decreases.

V. Conclusions and Discussion

This paper examines the contribution of cooperation on a diamond network with two and three conferencing relays. Our upper bound presented tighter results than the prior art for the two relays scenario. Furthermore, we proposed the three conferencing base stations scenario. A new coding technique has been developed, and upper bounds were derived, which were shown to outperform the cut-set bound for a regime of channel parameters.

Extending the technique proposed here to an arbitrary K relays setting is challenging due to the higher complexity of multiple paths and loops of information transmission in a cooperative network. However, it is manageable once some symmetry structure is assumed and restrictions on the cooperation links are imposed, and this is the focus of our future work.

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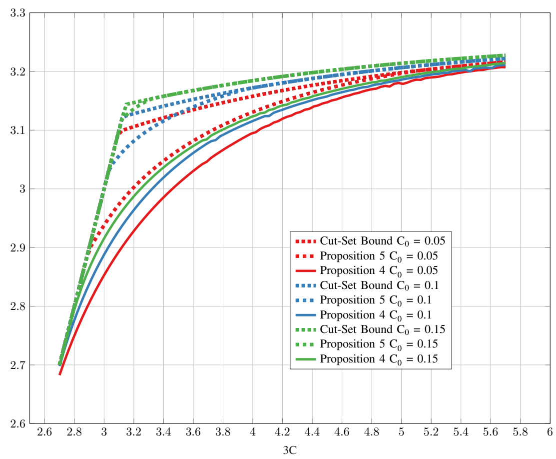

Fig. 4: Lower and upper bounds of the 3 relay setting from Props. 4 and 5, for various values of C_0 and $P = 1$ vs C.

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